Problems 7:
Ito differentiation rule

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Question 1

Consider the Ito formula for the differential of function $F(x, t)$:

$$dF(x,t) = \dot{F}\,dt + F'\,dx(t) + \frac{1}{2}F''\,dx^2(t)$$

What is the formula for dF in case of a differentiable process $x(t)$? (i.e. a process, for which $\dot{x}(t) = dx(t)/dt$ exists).

Question 2

Obtain differentials $dy = dF(x, t)$ for the following functions:

a) $y = \frac{x^2}{2} + e^{-at}$

b) $y = \ln x$

c) $y = e^x$

d) $y = \sin(x)$

Question 3

Complete the expressions for the differentials dy from the previous question for the process $x(t)$ described by the following stochastic differential equation:

a) $dx = \sqrt{2a}e^{-at/2}\,dw$

b) $\frac{dx}{x} = \mu\,dt + \sigma\,dw$

c) $dx = \mu x\,dt + \sigma\,dw$

d) $dx = \cos(x)\,dt + \sqrt{\frac{2}{\sin(x)}}\,dw$

Question 4

Is any of the processes $y(t)$, obtained in the previous question, differentiable at any t?