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Temporal Defeasible Reasoning

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Abstract. An argumentation system that allows temporal reasoning using the notions of *instant* and *interval* is presented. Previous proposals just considered either instants or intervals. A many-sorted logic is used to represent temporal knowledge at the monotonic level. The logic considers how to formalize knowledge about explicit temporal references, events, properties and actions. The argumentation system provides a non-monotonic layer in which to reason about the justification of truths in the system. The proposal is illustrated showing how to solve well-known problems of the literature.

Keywords: Defeasible reasoning; Knowledge representation; Temporal reasoning

1. Introduction

Argumentation systems (Chesñevar et al., 1998) are a way to formalize and implement defeasible reasoning, characterizing the skill that allows us to reason about a changing world where available information is incomplete or not very reliable. When new information is available, new reasons to obtain further conclusions or better reasons to sustain previous conclusions can be considered. But it could happen that some conclusions lose support. Through this inference dynamic, argumentation systems provide the ability to change conclusions according to the new information that comes to the system.

The conclusions obtained by the system are 'justified' through 'arguments' supporting their consideration. In addition, an argument could be seen as a 'defeasible proof' for a conclusion. The knowledge of new facts can lead one to prefer a conclusion to a previous one, or to consider a previous inference no longer correct. In particular, there could exist an argument for a conclusion C and a 'counter-argument', contradicting in some way the argument for C. An argument is a justification for a conclusion C if it is better than any other counter-argument

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for C. To establish the preference of an argument over the others, the definition of preference criteria is required. Although several preference methods are possible, one that is widely used is 'specificity': more specific information, i.e., better informed arguments, are preferred. It is important to highlight that argumentation systems emphasize the role of inference justification and the dialectical process related to reasoning activities. Argumentation systems proved to be very useful in a variety of areas (Carbogim et al., 2000), such as legal systems and negotiation in multi-agent frameworks, to name a few.

In our work the underlying argumentation system will be strongly related to that offered in Simari and Loui (1992). In Augusto and Simari (1994) and Augusto and Simari (1999) a temporal argumentation system, $\mathbb{L}(\mathscr{T})$, was defined using an ontology based on instants. Independently, in Ferguson and Allen (1994) and Ferguson (1995) an interval-based temporal argumentation system to reason in a multi-agent scenario was proposed.

The main goal of this article could be summarized as to offer a temporal argumentation system that improves previous proposals. We give an alternative temporal logic (Section 2) allowing interval as well as instant-based temporal references, which improves in several aspects the widely used Logic of Intervals (Allen and Ferguson, 1994). For example, there is no clear indication in Allen's proposal about syntax, semantics and rules of inference. One aim of this article is to offer an alternative monotonic logic for temporal reasoning providing such improvements. To fulfill this goal we make explicit syntax, semantics and inference rules of a many-sorted temporal logic based on previous work (Davidson, 1980; Allen, 1984; Gallier, 1987; Galton, 1990). After the presentation of the temporal logic we show how the notion of interval could be added to the instant-based system $\mathbb{L}(\mathscr{T})$ (Section 3). The persistency notion to be used (Section 4) borrows from previous proposals as those of (Haas, 1987) and (Schubert, 1994) but it is adapted to the argumentation framework we present. Examples are given (Section 5) to illustrate the behavior of the system to show how $\mathbb{L}(\mathbb{T})$ naturally embeds previous proposals made in Augusto and Simari (1994), Ferguson and Allen (1994), Ferguson (1995), Augusto and Simari (1999) and Augusto (2000). More details about our proposal can be found in Augusto (1998).

2. The Temporal Logic

In this section we will describe some basic features of the temporal language $\mathscr{L}^{\mathbb{T}}$ that is used by the argumentation system $\mathbb{L}(\mathbb{T})$. Restrictions of space force us to put aside some details. The reader will find in Augusto (1998, 2000) a more detailed explanation of the temporal monotonic layer of the proposal as well as proofs that $\mathscr{L}^{\mathbb{T}}$ subsumes the expressiveness of some well-known proposals of the literature (Allen, 1984; Bochman, 1990a; Bochman, 1990b; Benthem, 1991; Vila, 1994).

$\mathbb{L}(\mathbb{T})$ allows the interaction between both classic and defeasible temporal knowledge. Classic temporal knowledge will be specified through a many-sorted temporal logic. Its temporal language, $\mathscr{L}^{\mathbb{T}}$, permits reification over time, properties, events and actions. These have been considered in the literature of the area as key concepts in modeling a rational agent living in a dynamic world. Reification over more sorts of individuals is also allowed if the application demands it. The reason for choosing reification is that it bring us some advantages for knowledge representation and use. As Allen has pointed out (Allen, 1991), we need reification to efficiently represent information when we deal with an incomplete knowledge base. Also it is useful to efficiently handle the problem of distinguishing two individuals, e.g., if we need to know if two events are different or not. More motivations to use a many-sorted framework are (a) to have a clean way to specify each class of individuals under consideration, (b) to gain some computational efficiency restricting the set of individuals to look for during instantiations, and (c) that there are extensive studies of a many-sorted logic with functions and equality, giving syntax, semantics, proof theory and metatheoretical properties (Gallier, 1987).

We will take this previous work as a departing point. We shall concentrate on the extension of such a general framework to make it suitable for reasoning with temporal concepts connecting previous work in the literature to get a more precisely defined proposal. We define the well-formed formulas of the language $\mathscr{L}^{\mathbb{T}}$ using a BNF as follows.

Definition 1. Let s_i, s_j, s_m, s_n be sort names, the set of *well-formed formulas, wff, of* $\mathscr{L}^{\mathbb{T}}$ is defined as follows:

$$\begin{array}{lll}
\mathit{term}_{s_m} & ::= & \text{variable}_{s_m} \mid \text{constant}_{s_m} \mid \text{function_name}_{s_m}(\mathit{term_list}) \\
\mathit{term_list} & ::= & \mathit{term}_{s_i} \mid \mathit{term}_{s_j} \text{ , } \mathit{term_list} \\
\mathit{atomic_formula} & ::= & \text{predicate_name}(\mathit{term_list}) \\
\mathit{wff} & ::= & \mathit{atomic_formula} \mid (\mathit{term}_{s_n} \doteq_{s_n} \mathit{term}_{s_n}) \mid (\neg\ \mathit{wff}) \mid \\
 & & (\mathit{wff} \rightarrow \mathit{wff}) \mid (\mathit{wff} \wedge \mathit{wff}) \mid (\mathit{wff} \vee \mathit{wff}) \mid \\
 & & ((\exists_{s_n} \text{ variable}_{s_n})\ \mathit{wff}) \mid ((\forall_{s_n} \text{ variable}_{s_n})\ \mathit{wff})
\end{array}$$

Notation: Symbols of each sort, as $\doteq_s$ and $\forall_s$, are used only with symbols of the same sort. When it is clear from the context, we will omit the subscript to specify the intended sort. Also we take the convention of numbering just the axioms of the theory. Nested negations are ruled out, i.e. all formulas of the form $\neg\neg F$ will be considered as equivalent to F. We will use $\forall_{s_i} I_1, I_2 ...$ or $\exists_{s_i} I_1, I_2 ...$ instead of $\forall_{s_i} I_1 \forall_{s_i} I_2 ...$ and $\exists_{s_i} I_1 \exists_{s_i} I_2 ....$ We will also use $I_1 \gtrdot I_2, I_1 \lessdot I_2 \lessdot I_3, I_1 \sqsubseteq I_2 \sqsubseteq I_3$ instead of $I_2 \lessdot I_1, I_1 \lessdot I_2 \wedge I_2 \lessdot I_3, I_1 \sqsubseteq I_2 \wedge I_2 \sqsubseteq I_3$ respectively. We will proceed analogously when using '$<$' and symbols of other sorts.

We now give the reader an informal introduction to the language in order to provide a quick idea about what it looks like. All these notions will be explained in detail in the following sections. Some examples of temporal constants are: a date like 7-8-1991, symbols like Y_{1685} representing the year 1685 and numbers like 3600 that can be used to denote the number of seconds in an hour. Some examples of functions are *leap_year(A)* mapping a year into the constants *True* or *False* depending on *A* being a leap year or not and *seconds_year(A)* giving the number of seconds in a given year. Some examples of well-formed formulas of $\mathscr{L}^{\mathbb{T}}$ are:

$\text{Occurs\_}\mathit{during}(\mathit{born}(\mathit{jsbach}), Y_{1685})$
$\text{Do\_}\mathit{during}(\mathit{write}(\mathit{jsbach}, \mathit{magnificat}, \mathit{mi\_bemol}), Y_{1723})$
$\exists_{\mathscr{T}} i_1, i_2 \mathit{Precedes}(i_1, i_2)$
$\exists_{\mathscr{A}} a \exists_{\mathscr{E}} e \exists_{\mathscr{P}} p \exists_{\mathscr{T}} i(\text{Do}_{at}(a, i) \wedge \text{Occurs}_{at}(e, i+1) \rightarrow \text{Holds}_{at}(p, i+2))$
$\forall_{\mathscr{T}} i_1, i_2(\neg \mathit{Precedes}(i_1, i_2) \wedge \neg \mathit{Precedes}(i_2, i_1) \rightarrow \mathit{Simultaneous}(i_1, i_2))$
$\exists_{\mathscr{I}} I\ (\text{Occurs\_}\mathit{during}(\mathit{born}(\mathit{jsbach}), I) \wedge \text{Occurs\_}\mathit{during}(\mathit{born}(\mathit{gfhaendel}), I)$
$\wedge \text{Holds}(\mathit{good\_baroque\_year}, I))$

We will emphasize the use of formulas with temporal arguments but the language also allows us to use atemporal predicates like $Author(magnificat, jsbach)$ and $Choral\_piece(magnificat)$. In the following sections we give a short description of some sorts considered in the underlying temporal theory. The sorts to be described are: $\mathscr{E}_x$ for explicit time handling (Section 2.1), $\mathscr{E}_v$ for event-based time references (Section 2.2), $\mathscr{P}$ for properties (Section 2.3), and $\mathscr{A}$ for actions (Section 2.4). The sort $\mathscr{W}$ has all remaining objects of the world under formalization. Some of them can be axiomatized in a separate sort depending on the domain to be modeled and the range of problems to be solved.

2.1. The Temporal Domain

One of the well-known logics for temporal reasoning is the one presented in Allen (1984) where an interval-based ontology is adopted. The proposal banishes instants because in Allen's opinion they are of dubious existence and less practical than intervals (Allen and Hayes, 1989). Later, reasons were found to consider instants from the very beginning of the theory, e.g., in dealing with continuous change (Galton, 1990) and in dealing with the problem of truth change (Vila, 1994). See also Bochman (1990a) and Benthem (1991) for other proposals allowing both instants and intervals. The notion of instant also has practical advantages and sometimes representing interval-based knowledge using instants is so hard as it is in other cases to do it the other way around. Besides theoretical reasons there are practical considerations for adopting both instants and intervals as the basic temporal framework. In this work, instants will be identified with the smallest temporal references allowed in the system which in other works is called *chronos* or the *granularity* of the temporal system. Although sometimes 'instants' can be considered as having duration, we are assuming here that they have different properties from intervals and are treated in a different way. This is the reason why we do not accept *moments* as in Allen and Hayes (1989). Here we consider a theory where both are considered in the ontology but we explain how to start from the notion of instant defining intervals from them because it shows how to define the system in a modular way. This offers the possibility of starting from a simpler, instant-based framework as was done in Augusto and Simari (1994) and in a later stage to enlarge the system by adding the notion of interval as proposed here.

We will provide here an axiomatization of the sort for explicit temporal references, $\mathscr{E}_x$. Two kinds of elements, which in turn define subsorts, are considered in $\mathscr{E}_x$. We start its definition considering one of these subsorts, which are defined by 'instants'. Later we shall consider the other subsort, defined by 'intervals'. By an instant we mean the shortest temporal measure with respect to the granularity assumed on the system being modeled. An instant must not be considered here as durationless; instead it is the name of the unit of measure assumed in the system (which in some articles is called *chronos*). This is a point-based conception of time over which we shall later construct an interval-based structure. The subsort $\mathscr{T}$ is formalized in the structure $INS : \langle \mathscr{T}, < \rangle$ where $\mathscr{T}$ is a set of points of time termed 'instants' and $<: \mathscr{T} \times \mathscr{T}$ is an order relation. We usually denote members of $\mathscr{T}$ by i and its subscripts. The following axioms are valid in $\mathscr{T}$ and characterize an irreflexive (11), transitive (2) (hence asymmetric), non-ending

(3 and (4)) and discrete (5 and (6)) line (7) of time:

$$\forall i_1 \ \neg(i_1 < i_1) \tag{1}$$

$$\forall i_1, i_2, i_3 (i_1 < i_2 \wedge i_2 < i_3 \rightarrow i_1 < i_3) \tag{2}$$

$$\forall i_1 \ \exists i_2 (i_2 < i_1) \tag{3}$$

$$\forall i_1 \ \exists i_2 (i_1 < i_2) \tag{4}$$

$$\forall i_1, i_2 (i_1 < i_2 \rightarrow \neg \exists i_3 (i_1 < i_3 \wedge i_3 < i_2)) \tag{5}$$

$$\forall i_1, i_2 (i_2 < i_1 \rightarrow \neg \exists i_3 (i_2 < i_3 \wedge i_3 < i_1)) \tag{6}$$

$$\forall i_1, i_2 (i_1 < i_2 \vee i_2 < i_1 \vee i_1 \doteq i_2) \tag{7}$$

This excuses us from considering some characteristic problems of other structures but absent in discrete frameworks such as the intermingling problem (Galton, 1996) and the specification of the moment of change in a property (Vila, 1994). It is also important to notice we are not assuming the structure as isomorphic to $\mathbb{Z}$, keeping a first-order axiomatization of the temporal structure. We now define a notion of interval over INS as a subsort inside $\mathscr{E}_x$, which will be represented by means of $\mathscr{I}$.

The usual method to build intervals in similar frameworks is to consider them as a set of instants. Here we do not choose this method because of problems that arise in considering the occurrence of events associated to intervals in relation to its non-homogeneity property. That is to say, usually it is considered that if an event occurs in an interval conceived as a set of instants it also occurs in the set of instants that defines it. This conflicts with the non-homogeneity hypothesis over events. Since we are assuming events as non-homogeneous it is more adequate to associate an interval with a pair of instants considering it a unit. Notwithstanding, the points delimiting the interval allow us to do a kind of instant-based and constraint-based reasoning that has proved very useful in temporal reasoning (Meiri, 1992).

Definition 2. We will call an *interval* each member of

$$\mathscr{I} = \{[i_1, i_2] \in \mathscr{T} \times \mathscr{T} | i_1 < i_2\}$$

We shall also consider the function $int : \mathscr{T} \times \mathscr{T} \rightarrow \mathscr{I}$ where

$$int(i_1, i_2) =_{def} [i_1, i_2] \text{ if } i_1 < i_2$$

Notation: We shall change the usual parentheses associated with ordered pairs to brackets to align them to the usual appearance in the temporal reasoning literature. We shall also usually denote intervals by I and its subscripts.

As can be noticed, we are discarding ‘punctual intervals’, i.e., intervals of the form $[i, i]$. This is because, being simultaneously an instant and an interval, both their meaning and their set of properties would be ambiguous.

Definition 3. We will consider the total functions $begin, end : \mathscr{I} \rightarrow \mathscr{T}$ that give us for each interval their beginning and ending points respectively:

$$begin([i_1, i_2]) =_{def} i_1 \quad \text{and} \quad end([i_1, i_2]) =_{def} i_2$$

Now we could consider a structure $INT : \langle \mathscr{I}, \prec, \sqsubseteq \rangle$ where $\mathscr{I}$ is a set of intervals

Table 1. Interval–interval relations.

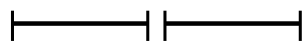
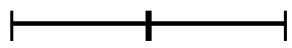
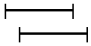
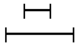
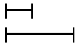
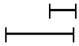
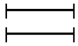
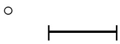
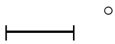
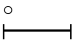
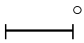
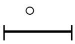
Relation	Conditions	Relation	Conditions
BEFORE(X,Y)		MEETS(X,Y)	
OVERLAP(X,Y)		DURING(X,Y)	
STARTS(X,Y)		FINISHES(X,Y)	
EQUAL(X,Y)			

Table 2. Point–interval relations.

Relation	Conditions	Relation	Conditions
Precedes(*i*,*I*)		*Follows*(*i*,*I*)	
Start(*i*,*I*)		*Ends*(*i*,*I*)	
Divides(*i*,*I*)			

and $\sqsubseteq, \lessdot \subseteq \mathscr{I} \times \mathscr{I}$ are the relations 'previous to' and 'subinterval', defined as follows:

$$I \lessdot I' =_{def} \{I, I' | I = [i_1, i_2], I' = [i'_1, i'_2] \text{ and } i_2 < i'_1\}$$

$$I \sqsubseteq I' =_{def} \{I, I' | I = [i_1, i_2], I' = [i'_1, i'_2], i'_1 \leqslant i_1 \text{ and } i_2 \leqslant i'_2\}$$

Also we will use the following definition:

$$I_1 \cdot\!=\!\cdot\ I_2 =_{def} begin(I_1) \doteq begin(I_2) \wedge end(I_1) \doteq end(I_2)$$

Because of the temporal entities introduced, we can now define a set of well-known relations in the literature as those between intervals defined by Hamblin (1972) and later adopted by Allen (1984) and those between points and intervals (Meiri, 1992). That is to say, interval relations (see Table 1) BEFORE, MEETS, OVERLAPS, BEGINS, DURING, FINISHES, EQUALS, their inverses and the following relations between points and intervals (see Table 2): *precedes*, *start*, *divides*, *ends*, *follows*, can be defined in $\mathscr{T}$.

As Allen and Hayes (1985) have shown, all these interval relations can be defined from MEETS. However, we will define them all to show how direct it is in our framework and to allow future citations of each relation:

$$\begin{array}{lll}
MEETS(I_1, I_2) & =_{def} & end(I_1) \doteq begin(I_2) \\
BEFORE(I_1, I_2) & =_{def} & end(I_1) < begin(I_2) \\
OVERLAPS(I_1, I_2) & =_{def} & begin(I_1) < begin(I_2) < end(I_1) < end(I_2) \\
BEGINS(I_1, I_2) & =_{def} & begin(I_1) \doteq begin(I_2) \wedge end(I_1) < end(I_2) \\
DURING(I_1, I_2) & =_{def} & begin(I_2) < begin(I_1) \wedge end(I_1) < end(I_2) \\
FINISHES(I_1, I_2) & =_{def} & begin(I_2) < begin(I_1) \wedge end(I_1) \doteq end(I_2) \\
EQUALS(I_1, I_2) & =_{def} & I_1 \cdot\!\!=\!\!\cdot\ I_2 \\
Precedes(i, I) & =_{def} & i < begin(I) \\
Start(i, I) & =_{def} & i \doteq begin(I) \\
Divides(i, I) & =_{def} & begin(I) < i < end(I) \\
Ends(i, I) & =_{def} & i \doteq end(I) \\
Follows(i, I) & =_{def} & end(I) < i
\end{array}$$

Our $\lessdot$, and $\cdot\!\!=\!\!\cdot$ are Allen's *BEFORE* and *EQUAL*, while our $\sqsubseteq$ is split into *BEGINS, FINISHES, DURING* and *EQUALS*. We could also obtain similar theorems to Axiom 7 stating that each pair of intervals is in one of the 13 mutually exclusive relations proposed by Allen. Analogously, each point–interval pair is in exactly one of the mutually exclusive relations between points and intervals. We skip the proofs for the sake of brevity.

We can identify some general properties (Benthem, 1991) about the structure $\mathscr{T}$. It satisfies *Symmetry*, i.e., seeing to the future and the past is not different, *Connection*, i.e., all pairs of points are related, and *Homogeneity*, i.e., all points are of the same quality. One needs to look at the structure as a whole to justify these properties. For example, symmetry can be verified through axioms for the structure INS that provides equal characterization in both directions of the temporal line. All instants are obviously connected through the comparison relation, $<$. The structure is also homogeneous because there is no such distinguished points like 'a first moment', 'a last moment' or the concept of 'now'. Consequently, it can also be seen that *Connection*, *Symmetry* and *Homogeneity* are valid for the structure $\mathscr{I}$.

It is important to highlight that we left open the possibility of taking advantage of previous research where algorithms were proposed to solve constraint problems (Meiri, 1992) involving instants and intervals in every possible combination. These algorithms will give us more efficiency in the temporal constraint solver than when we are forced to do constraint reasoning in a purely interval-based framework (Vilain et al., 1989).

2.2. Events

Our proposal also considers a way to reason about change without explicit time. For this purpose we use a framework similar to that adopted for the sort $\mathscr{E}_x$, this time splitting $\mathscr{E}_v$ into two subsorts $\mathscr{N}$ and $\mathscr{D}$ for non-durative (punctual) and durative events respectively.

An *event structure* $PUN : (\mathscr{N}, <_{\mathscr{E}})$ is considered where $\mathscr{N}$ is a set of punctual events and $<_{\mathscr{E}} \subseteq \mathscr{N} \times \mathscr{N}$ is a binary order relation. We assume the same temporal structure from the event-based perspective. The goal is to enhance interaction and cooperation between sorts $\mathscr{E}_v$ and $\mathscr{E}_x$, which are used as complementary and cooperative temporal perspectives. This interaction is achieved through axioms to be given at the end of this section. The following axioms hold in the structure

PUN, defining a linear, non-ending and discrete structure of instantaneous events:

$$\forall e \neg(e <_{\mathscr{E}} e) \quad (8)$$

$$\forall e, e', e''(e <_{\mathscr{E}} e' \wedge e' <_{\mathscr{E}} e'' \rightarrow e <_{\mathscr{E}} e'') \quad (9)$$

$$\forall e \exists e'(e' <_{\mathscr{E}} e) \quad (10)$$

$$\forall e \exists e'(e <_{\mathscr{E}} e') \quad (11)$$

We consider a relation of *punctual simultaneity* represented by $S_p \subseteq \mathscr{N} \times \mathscr{N}$ where

$$e_1 S_p e_2 =_{def} \neg(e_1 <_{\mathscr{E}} e_2 \vee e_2 <_{\mathscr{E}} e_1)$$

As linearity does not follow from the previous axioms then we state the following:

$$\forall e, e', e''(eS_pe' \wedge e'S_pe'' \rightarrow eS_pe'') \quad (12)$$

It can be proved (Augusto, 1998) that S_p defines an equivalence relation over $\mathscr{N}$. Analogously to the sort $\mathscr{T}$ we will assume:

$$\forall e_1, e_2(e_1 <_{\mathscr{E}} e_2 \rightarrow \neg\exists e_3(e_1 <_{\mathscr{E}} e_3 \wedge e_3 <_{\mathscr{E}} e_2)) \quad (13)$$

$$\forall e_1, e_2(e_2 <_{\mathscr{E}} e_1 \rightarrow \neg\exists e_3(e_2 <_{\mathscr{E}} e_3 \wedge e_3 <_{\mathscr{E}} e_1)) \quad (14)$$

$$\forall e_1, e_2(e_1 <_{\mathscr{E}} e_2 \vee e_2 <_{\mathscr{E}} e_1 \vee e_1 S_p e_2) \quad (15)$$

The *event structure DUR* : ($\mathscr{D}$, $B_{\mathscr{E}}$, $E_{\mathscr{E}}$) comprises a set $\mathscr{D}$ of durative events over which two functions are defined, $B_{\mathscr{E}}$, $E_{\mathscr{E}}$: $\mathscr{D} \rightarrow \mathscr{N}$, by means of which we could obtain punctual events associated with the beginning and end of a durative event. Other useful relations such as *durative simultaneity*, *overlapping events* and *abutting* are easily obtained.

Using the recently defined notion of event we could define the temporal notions associated with explicit time references.

Definition 4. Let PUN : ($\mathscr{N}$, $<_{\mathscr{E}}$) be a structure of punctual events. As the simultaneity relation defines an equivalence relation over $\mathscr{N}$, we can identify an 'instant' with each simultaneity class so defined over $\mathscr{N}$. Also we will consider a function *e_instant* : $\mathscr{N} \rightarrow \mathscr{T}$ that returns a name of an instant associated with the simultaneity class to which a given punctual event belongs.

Durative events can be defined as those represented by a *chain of events*. The sort $\mathscr{E}_v$ is defined as a sort of mirror image of $\mathscr{E}_x$ as much as possible. Naturally they offer different means to represent knowledge but both are intended to reflect the same conception of time. This allows to define functions to transfer temporal knowledge between the sorts when possible (Augusto, 1998; Augusto, 2000); for example, a set of functions giving the explicit beginning or ending time associated with a durative event. This could be used with a set of connecting axioms allowing to transfer knowledge from one sort to another and together they provide new means to reason about change. This gives us a way to draw conclusions and reasoning strategies that could not be obtained if we consider each sort as an isolated source of information.

We now make explicit the aforementioned axioms connecting the sorts $\mathscr{E}_v$ and $\mathscr{E}_x$. Let $\text{Occurs}_{at}(e, i)$ and $\text{Occurs}_{on}(e, I)$ denote the occurrence of an event e in the moment i or interval I respectively. They are related in the following sense:

$$\text{Occurs}_{on}(e, I) =_{def} \forall_{\mathscr{T}}\ i\ (\text{In}(i, I) \rightarrow \neg\text{Occurs}_{at}(e, i))$$

where $\text{In}(i, I) =_{def}\ Start(i, I) \vee Divides(i, I) \vee Ends(i, I)$. This axiom reflects event non-homogeneity, i.e. the occurrence of an event in an interval implies it does not occur inside the interval. For example, if the event of a pencil being passed from one hand to another occurs in an interval $[a, b]$, it cannot be said to occur in $[c, d]$ such that $a < b \leqslant c \leqslant d$ or $a \leqslant b \leqslant c < d$. The event takes the whole interval.

The following axioms provide a means to transfer knowledge between the two ways of representing temporal order:

$$\forall_{\mathscr{N}}\ e_1, e_2\ \forall_{\mathscr{T}}\ i_1, i_2\ (\text{Occurs}_{at}(e_1, i_1) \wedge \text{Occurs}_{at}(e_2, i_2) \wedge e_1 <_{\mathscr{E}} e_2 \rightarrow i_1 < i_2) \tag{16}$$

$$\forall_{\mathscr{N}}\ e_1, e_2\ \forall_{\mathscr{T}}\ i_1, i_2\ (\text{Occurs}_{at}(e_1, i_1) \wedge \text{Occurs}_{at}(e_2, i_2) \wedge i_1 < i_2 \rightarrow e_1 <_{\mathscr{E}} e_2) \tag{17}$$

Theorems with similar appearance and purpose can be extracted from previous axioms using a notion of temporal order suitable for durative events (Augusto, 1998, 2000). Finally, we consider 'weak negation' over durative events in the following sense:

$$\neg\text{Occurs}_{on}(e, I) =_{def} \exists_{\mathscr{T}}\ i\ (\text{In}(i, I) \wedge \neg\text{Occurs}_{at}(e, i))$$

2.3. Properties

For the representation of properties we will consider predicates like those introduced by Galton (1990) who states: $\text{Holds}_{at}(p, i)$, $\text{Holds}_{at} \subseteq \mathscr{P} \times \mathscr{T}$, and $\text{Holds}_{on}(p, I)$, $\text{Holds}_{on} \subseteq \mathscr{P} \times \mathscr{I}$, denoting that p is a property that is true in the moment i or interval I respectively. For example, we will use something like $A(x)$ to assert that x possesses the property A. Holds_{on} and Holds_{at} are related in the following way:

$$\text{Holds}_{on}(p, I) =_{def} \forall_{\mathscr{T}}\ i\ (\text{In}(i, I) \rightarrow \text{Holds}_{at}(p, i))$$

From the previous definition we get the following theorems about homogeneity of properties over an interval. This means that if a property holds in an interval then it also holds in any of its subintervals. For example, if a door was green during a week it was also green each day of that week.

$$\forall_{\mathscr{T}}\ i\ \forall_{\mathscr{I}}\ I\ (\text{Holds}_{on}(p, I) \wedge \text{In}(i, I) \rightarrow \text{Holds}_{at}(p, i)$$

$$\forall_{\mathscr{I}}\ I, I'\ (\text{Holds}_{on}(p, I) \wedge I' \sqsubseteq I) \rightarrow \text{Holds}_{on}(p, I'))$$

We consider 'weak negation' of properties over intervals that can be obtained directly from the negation of the previous definition:

$$\neg\text{Holds}_{on}(p, I) =_{def} \exists_{\mathscr{T}}\ i\ (\text{In}(i, I) \wedge \neg\text{Holds}_{at}(p, i))$$

In what follows we will use a relation $\text{Changes}(e, p) : \mathscr{E}_v \times \mathscr{P}$ denoting that e is an event which, every time it occurs, causes a change of the property p. The following axiom states that whenever a property changes its truth value it is because an event occurred:

$$\forall_{\mathscr{P}} p \ \forall_{\mathscr{T}} i((\text{Holds}_{at}(p,i) \land \neg\text{Holds}_{at}(p,i+1)) \lor (\neg\text{Holds}_{at}(p,i) \land \text{Holds}_{at}(p,i+1)) \rightarrow \exists_{\mathscr{E}v}\ e\ (\text{Changes}(e,p) \land [\exists_{\mathscr{T}} i'(\text{Occurs}_{at}(e,i') \land i' < i+1) \lor \exists_{\mathscr{I}} I(\text{Occurs}_{on}(e,I) \land \ begin(I) \ < i+1)]) \tag{18}$$

2.4. Actions

In some contexts it is difficult to differentiate one action from the event that it causes, e.g., John's flipping a switch. The reader could consider whether it was really necessary to have another sort for action since its consideration leads sometimes to possibly artificial differentiations between them. In this article we will follow the hypothesis that this feature is convenient to the option of not being allowed to distinguish them when it is needed. We will consider that every action is performed by an agent:

$$\forall_{\mathscr{A}}\ a\ \exists_{\mathscr{W}}\ g\ \text{Agent}(a,g) \tag{19}$$

Agents are assumed to be individuals in sort $\mathscr{W}$. This is left unspecified, allowing different personalities to be encoded. As with previous sorts we consider predicates Do_{at}, denoting instantaneous actions like snapping the fingers or blinking the eyes, and Do_{on} for durative actions, as raising the arm. More axioms including actions but in relation to events and causality are considered in the next section.

2.5. Causality

This section is devoted to a short introduction to some of the assumptions in the monotonic layer concerning the relation between causality and temporality. We focus in some basic axioms imposing general constraints on predicates related to causality. They must be supplemented with other axioms bringing knowledge about particularities of causality between specific events and actions. The reader could find a more detailed explanation in Augusto (1998).

We will consider first action causality and as a first hypothesis we will suppose that an event cannot be previous to the action that produces it. We consider then two situations. One option is that the beginning points could be simultaneous, like perceiving the color of an object when we look at it or producing sounds while drawing a stick over the strings of an instrument. Also they could be overlapping as when somebody pushes an object during a period until it collides with another object. Another example is a law that is promulgated on a certain date but whose effects could begin later.

We will also assume that an event produced by an action can finish before the end of the action that produces it. One scenario in which this could happen is when an action produces an instantaneous event. For example, my action of spreading water of a glass causes the event of starting to spread the water or the event of the first contact of water with the floor. The effect of an action can also cease before it is expected, e.g., when somebody passes a bow across the strings of an instrument and after some time the sound ceases because a string breaks.

Let us suppose we use the predicate Acause as a relation Acause $\subseteq \mathscr{A} \times \mathscr{E}_v$ denoting that an action causes an event occurrence. We could summarize the previous ideas through the following axioms stating that each event caused by an agent occurs through the performance of previous or contemporary actions:

$$\begin{aligned}\forall_{\mathscr{N}}\ e\ \forall_{\mathscr{T}}\ i_e\ \exists_{\mathscr{A}}\ a\ (\mathrm{Occurs}_{at}(e,i_e) \wedge\ \mathrm{Acause}(a,e) \rightarrow \\ [\exists_{\mathscr{T}}\ i_a\ (\mathrm{Do}_{at}(a,i_a) \wedge i_a \leqslant i_e) \vee \\ \exists_{\mathscr{I}}\ I_a\ (\mathrm{Do}_{on}(a,I_a) \wedge\ begin(I_a) \leqslant i_e)])\end{aligned} \tag{20}$$

$$\begin{aligned}\forall_{\mathscr{D}}\ e\ \forall_{\mathscr{I}}\ I_e\ \exists_{\mathscr{A}}\ a\ (\mathrm{Occurs}_{on}(e,I_e) \wedge\ \mathrm{Acause}(a,e) \rightarrow \\ [\exists_{\mathscr{T}}\ i_a\ (\mathrm{Do}_{at}(a,i_a) \wedge\ i_a \leqslant begin(I_e)) \vee \\ \exists_{\mathscr{I}}\ I_a\ (\mathrm{Do}_{on}(a,I_a) \wedge\ (begin(I_a) \leqslant begin(I_e)))])\end{aligned} \tag{21}$$

As with action causation we introduce a predicate, Ecause $\subseteq \mathscr{E}_v \times \mathscr{E}_v$, denoting that there exists a correlation between two event occurrences. Actually, this is a simplification of the problem because it may be argued that this can be considered as a relation Ecause $\subseteq \mathscr{P} \times \mathscr{E}_v \times \mathscr{E}_v$. The reason that could be given is that properties are needed to allow the causing event to occur. Similar, but weaker, arguments could be given to include the properties that change as part of the effect. We opted to simplify this point assuming that there are rules in the knowledge base to link properties with events in an appropriate way to specify the dependence of events with its associated properties. Similarly to Acause we consider the following axioms on event causation stating that events can be the cause of other previous or contemporary events:

$$\begin{aligned}\forall_{\mathscr{N}}\ e\ \forall_{\mathscr{T}}\ i\ \exists_{\mathscr{E}_v}\ e'\ (\mathrm{Occurs}_{at}(e,i) \wedge\ \mathrm{Ecause}(e',e) \rightarrow \\ [\exists_{\mathscr{T}}\ i'\ (\mathrm{Occurs}_{at}(e',i') \wedge i' \leqslant i) \vee \\ \exists_{\mathscr{I}}\ I'\ (\mathrm{Occurs}_{on}(e',I') \wedge\ begin(I') \leqslant i)])\end{aligned} \tag{22}$$

$$\begin{aligned}\forall_{\mathscr{D}}\ e\ \forall_{\mathscr{I}}\ I\ \exists_{\mathscr{E}_v}\ e'\ (\mathrm{Occurs}_{on}(e,I) \wedge\ \mathrm{Ecause}(e',e) \rightarrow \\ [\exists_{\mathscr{T}} i'(\mathrm{Occurs}_{at}(e',i') \wedge\ i' \leqslant begin(I))\ \vee \\ \exists_{\mathscr{I}} I'\ (\mathrm{Occurs}_{on}(e',I') \wedge\ begin(I') \leqslant\ begin(I))])\end{aligned} \tag{23}$$

2.6. Inference Rules

We give in this section a set of inference rules borrowed from Gallier's proposal for a many-sorted logic (Gallier, 1987). This is a Gentzen system for many-sorted logics with equality. In doing so we will use the following notion of *sequent*.

Definition 5. A *sequent* is a pair (Γ, Δ) of finite (possibly empty) sequences $\Gamma = \langle A_1, \ldots, A_m \rangle, \Delta = \langle B_1, \ldots, B_m \rangle$ of propositions. We will write them as $\Gamma \vdash \Delta$ for clarity.

Let Δ, Γ, Θ denote arbitrary sequences of formulas and A, B denote formulas, then the rules of the sequent calculus are the following:

$$\frac{\Gamma,A,B,\Delta\vdash\Theta}{\Gamma,A\wedge B,\Delta\vdash\Theta}(\wedge - L) \qquad \frac{\Gamma\vdash\Delta,A,\Theta \quad \Gamma\vdash\Delta,B,\Theta}{\Gamma\vdash\Delta,A\wedge B,\Theta}(\wedge - R)$$

$$\frac{\Gamma,A,\Delta\vdash\Theta \quad \Gamma,B,\Delta\vdash\Theta}{\Gamma,A\vee B,\Delta\vdash\Theta}(\vee - L) \qquad \frac{\Gamma\vdash\Delta,A,B,\Theta}{\Gamma\vdash\Delta,A\vee B,\Theta}(\vee - R)$$

$$\frac{\Gamma,\Delta\vdash A,\Theta \quad B,\Gamma,\Delta\vdash\Theta}{\Gamma,A\rightarrow B,\Delta\vdash\Theta}(\rightarrow -L) \qquad \frac{A,\Gamma\vdash B,\Delta,\Theta}{\Gamma\vdash\Delta,A\rightarrow B,\Theta}(\rightarrow -R)$$

$$\frac{\Gamma,\Delta\vdash A,\Theta}{\Gamma,\neg A,\Delta\vdash\Theta}(\neg - L) \qquad \frac{A,\Gamma\vdash\Delta,\Theta}{\Gamma\vdash\Delta,\neg A,\Theta}(\neg - R)$$

In the quantifier rules below, x is any variable of sort s and y is any variable of sort s free for x in A and not free in A, unless $y = x$. The term t is any term of sort s free for x in A. Of course, in each substitution $[t/x]$ of a variable x by a term t, the sort of the term must be the same as the sort of the variable.

$$\frac{\Gamma,A[t/x],\forall_{s_i}xA,\Delta\vdash\Theta}{\Gamma,\forall_{s_i}xA,\Delta\vdash\Theta}(\forall - L) \qquad \frac{\Gamma\vdash\Delta,A[y/x],\Theta}{\Gamma\vdash\Delta,\forall_{s_i}xA,\Theta}(\forall - R)$$

$$\frac{\Gamma,A[y/x],\Delta\vdash\Theta}{\Gamma,\exists_{s_i}xA,\Delta\vdash\Theta}(\exists - L) \qquad \frac{\Gamma\vdash\Delta,A[t/x],\exists_{s_i}xA,\Theta}{\Gamma\vdash\Delta,\exists_{s_i}xA,\Theta}(\exists - R)$$

In both $\forall - R$ and $\exists - L$ rules, the variable y does not occur free in the lower sequent. For equality, let Δ, Γ, Θ denote arbitrary sequences of formulas (possibly empty) and let $t, \ldots, t_1, \ldots, t_n, t'_1, \ldots, t'_n$ denote arbitrary terms. For every sort s_i and for every term t of sort s_i we have

$$\frac{\Gamma, t \doteq_{s_i} t \vdash \Delta}{\Gamma \vdash \Delta}$$

For each function symbol f of arity $(s_1, \ldots, s_n, s)$ and any terms $t_1, \ldots, t_n$, $t'_1, \ldots, t'_n$ such that t_i are of sort s_i:

$$\frac{\Gamma, (t_1 \doteq_{s_1} t'_1) \wedge \ldots \wedge (t_n \doteq_{s_n} t'_n) \rightarrow (f(t_1, \ldots, t_n) \doteq_s f(t'_1, \ldots, t'_n)) \vdash \Delta}{\Gamma \vdash \Delta}$$

For each predicate symbol P (including $\doteq_{s_i}$) of arity $(s_1, \ldots, s_n)$ and any terms $t_1, \ldots, t_n, t'_1, \ldots, t'_n$ such that t_i are of sort s_i:

$$\frac{\Gamma, ((t_1 \doteq_{s_1} t'_1) \wedge \ldots \wedge (t_n \doteq_{s_n} t'_n) \rightarrow P(t_1, \ldots, t_n)) \rightarrow P(t'_1, \ldots, t'_n) \vdash \Delta}{\Gamma \vdash \Delta}$$

Gallier (1987) gives a detailed exposition of the metatheoretical properties and the proof procedure associated with a many-sorted logic with these rules of inference.

It can be observed that the previous rules of inference give us a way to infer when two individual references could be considered as referring to the same object. This provides the same general theory of equality for all sorts. However, it is interesting to consider particularities associated with the individuation of members of each sort. This is of particular interest in artificial intelligence applications where knowledge about the world is supposed to be usually poor and we need to find other means to get implicit knowledge from previous explicit knowledge. The reader will find in Augusto (1998, 2000) some more specific proposals to individuate objects depending on the sort to which they are supposed to belong.

2.7. Semantics

We have considered a many-sorted algebra-based semantics (Gallier, 1987) for $\mathscr{L}^{\mathbb{T}}$ as follows. The different sorts are carriers and each sort s_k has its own function mapping terms, possibly from different sorts, to terms in the sort s_k, i.e., $f : s_i \times \ldots \times s_j \rightarrow s_k$. A special Boolean sort is considered, $\mathscr{B}$, with the constants *true* and *false* as elements. Boolean classical operators like $\wedge, \vee, \rightarrow$ are regarded as functions of type $f : \mathscr{B} \times \mathscr{B} \rightarrow \mathscr{B}$ and $\neg$ as $f : \mathscr{B} \rightarrow \mathscr{B}$. Each predicate $P(t1_{s1}, \ldots, tn_{sn})$ has associated a function mapping terms $t1_{s1}, \ldots, tn_{sn}$ from sorts $s1, \ldots, sn$ to $\mathscr{B}$. In particular, we could interpret the symbol $\doteq$ in this way.

Let M be the many-sorted algebra associated to $\mathscr{L}^{\mathbb{T}}$ as explained above. Each sub-carrier of M is called M_s for each s a sort in $\mathscr{L}^{\mathbb{T}}$. We could consider a set $V = \cup_s v_s$ formed with each set v_s of variables from the sort s. An assignment $v : V \rightarrow M$ can be defined using assignments $v_s : V_s \rightarrow M_s$ for each sort. We will use $M \models A[v]$ to mean that the assignment v satisfies the formula A in M. An interpretation function for terms, t, can be defined recursively in the usual way. If A and B are formulas of $\mathscr{L}^{\mathbb{T}}$, t_1 and t_2 are terms in $\mathscr{L}^{\mathbb{T}}$ then

$M \models (t_1 \doteq t_2)[v]$ iff $t(t_1) = t(t_2)$
$M \models (\neg A)[v]$ iff not $M \models A[v]$
$M \models (A \wedge B)[v]$ iff $M \models A[v]$ and $M \models B[v]$
$M \models (A \vee B)[v]$ iff $M \models A[v]$ or $M \models B[v]$
$M \models (A \rightarrow B)[v]$ iff not $M \models A[v]$ or $M \models B[v]$
$M \models (\forall_s x_i A)[v]$ iff $M \models (A[a/x_i])[v]$ for every $a \in M_s$
$M \models (\exists_s x_i A)[v]$ iff $M \models (A[a/x_i])[v]$ for some $a \in M_s$

Definition 6. We can say that M *satisfies* A *with* v if and only if $M \models A[v]$. A formula is *satisfiable in* M if and only if there is some assignment v such that $M \models A[v]$. A is *satisfiable* if and only if there is some M such that A is satisfiable.

Definition 7. A formula is *valid in* M, $M \models A$, if and only if $M \models A[v]$ for all v. In such case, M is called a *model* of A. A formula is valid, $\models A$, if it is valid in each structure M.

These definitions can be extended in the usual way to consider satisfiability, validity and models of a set of formulas. Then, given a set Γ of formulas and a formula B we will say that B *is a logical consequence of* Γ, $\Gamma \models B$, if and only if for every structure M associated to a language $\mathscr{L}^{\mathbb{T}}$ and for every assignment v: if $M \models A[v]$ for every formula $A \in \Gamma$ then $M \models B[v]$.

3. The Extended Argumentation System

In this section a formal system for temporal defeasible reasoning called $\mathbb{L}(\mathbb{T})$ will be defined. It is intended as the representation of an intelligent agent $\mathscr{A}^{\mathbb{T}}$ capable of carrying on such activities as non-monotonic and temporal reasoning. It represents an update of the one offered through Augusto and Simari (1994, 1999), allowing us to use the notion of interval as another way of referencing time. In this proposal explicit temporal references in each term of the language can be made by way of either instants or intervals. In the first case it will be a point in the temporal line and in the latter case an ordered pair defined by two points as in $[i_1, i_2]$.

3.1. The Knowledge Base

The temporal knowledge base is organized as presented in Simari and Loui (1992) for the atemporal case. We will consider a *temporal context* as a finite set $\mathscr{K}^{\mathbb{T}}$ of well-formed and consistent formulas of $\mathscr{L}^{\mathbb{T}}$. $\mathscr{K}^{\mathbb{T}}$ contains the non-defeasible knowledge of $\mathscr{A}^{\mathbb{T}}$. $\mathscr{K}^{\mathbb{T}}$ is formed by two sets, one set of facts $\mathscr{K}_G^{\mathbb{T}}$ (general knowledge) and one set of rules $\mathscr{K}_P^{\mathbb{T}}$ (particular knowledge), where $\mathscr{K}_P^{\mathbb{T}} \cup \mathscr{K}_G^{\mathbb{T}} = \mathscr{K}^{\mathbb{T}}$ and $\mathscr{K}_P^{\mathbb{T}} \cap \mathscr{K}_G^{\mathbb{T}} = \emptyset$. $\mathscr{K}_P^{\mathbb{T}}$ represents the safe knowledge of the world like the existence of individuals or well-known properties of objects at a given time, e.g. a triangle has three sides. $\mathscr{K}_G^{\mathbb{T}}$ represents general laws such as mathematical properties or the axioms defining the ontology.

Example 1. Let us suppose that capital letters represent predicate names, the first non-capital letters of the alphabet are constants, last non-capital letters of the alphabet are variables, and i is a variable for instants. Then the following is an example of the safe part of a knowledge base:

$$\mathscr{K}^{\mathbb{T}} = \underbrace{\{A(a,2), B(a,b,2), E(a,e,2)\}}_{\mathscr{K}_P^{\mathbb{T}}} \cup \underbrace{\{\neg C(x,i) \rightarrow \neg D(x,[i+1,i+3])\}}_{\mathscr{K}_G^{\mathbb{T}}}$$

Definition 8. $\Delta^{\mathbb{T}}$ is a finite set of *temporal defeasible rules* representing knowledge that $\mathscr{A}^{\mathbb{T}}$ is prepared to take unless it possesses counter-evidence. Rules in $\Delta^{\mathbb{T}}$ have the form $\alpha \succ\!\!- \beta$, where α and β are sets of literals of $\mathscr{L}^{\mathbb{T}}$. $\Delta^{\mathbb{T}\downarrow}$ will denote the set of basic instances of members of $\Delta^{\mathbb{T}}$.

Example 2. We can consider the following set of temporal defeasible rules:

$$\Delta^{\mathbb{T}} = \{ \quad \begin{aligned} & A(x,i) \wedge B(x,y,i) \wedge E(x,z,i) \succ\!\!- \neg C(x,i+1), \\ & A(x,i) \wedge B(x,y,i) \succ\!\!- C(x,i+1), \\ & C(x,i+1) \succ\!\!- D(x,[i+2,i+6]) \end{aligned} \quad \}$$

3.2. Temporal Arguments

Arguments in $\mathbb{L}(\mathbb{T})$ are built using facts of $\mathscr{K}_P^{\mathbb{T}}$ and rules of $\mathscr{K}_G^{\mathbb{T}}$ and $\Delta^{\mathbb{T}\downarrow}$. Sometimes it is useful to think of $\mathscr{K}_P^{\mathbb{T}}$ as being organized into subsets called *snapshots*, for each instant i mentioned in it. $\mathscr{K}^{\mathbb{T}}$ and $\Delta^{\mathbb{T}}$ form the knowledge base.

Definition 9. The pair $(\mathscr{K}^{\mathbb{T}}, \Delta^{\mathbb{T}})$ will be called the *temporal defeasible structure*, where $\mathscr{K}^{\mathbb{T}}$ is a temporal context and $\Delta^{\mathbb{T}}$ is a finite set of temporal defeasible rules.

Definition 10. Let $\Gamma^{\mathbb{T}} = \{A_1, A_2, \dots A_n\}$, where each A_i is an element of $\mathscr{K}^{\mathbb{T}}$ or $\Delta^{\mathbb{T}\downarrow}$. We will consider the meta-meta-relation '$\mid\!\sim$', *temporal defeasible consequence*, between $\Gamma^{\mathbb{T}}$ and a literal A. We will say that $\Gamma^{\mathbb{T}} \mid\!\sim A$ if and only if there exists $B_1, \dots, B_n$ such that $A = B_n$ and for every i, $B_i \in \Gamma^{\mathbb{T}}$ or B_i is a direct consequence of preceding elements in the sequence using rules of temporal inference. $\Delta^{\mathbb{T}\downarrow}$ is considered as a set of material implications.

We assume a set of Gentzen-style sequent rules (Gallier, 1987) but as the reader can appreciate the system was defined in such a way that they could be changed.

We are now going to present one of the key notions in this kind of system: that of a temporal argument.

Definition 11. Given a defeasible temporal structure $(\mathcal{K}^{\mathbb{T}}, \Delta^{\mathbb{T}})$, a subset A of $\Delta^{\mathbb{T}\downarrow}$ is a *temporal argument* for a temporal literal h $\langle A, h\rangle$, if and only if : 1) $\mathcal{K}^{\mathbb{T}} \cup A \mid\!\sim h$, 2) $\mathcal{K}^{\mathbb{T}} \cup A \not\mid\!\sim \bot$ and 3) there is no $A' \subset A$ such that $\mathcal{K}^{\mathbb{T}} \cup A' \mid\!\sim h$.

This defines an argument as the minimal set of defeasible rules consistent with the safe knowledge allowing to infer the supported thesis. A related notion that will be useful later is that of a sub-argument.

Example 3. We can consider the following arguments A_1 and A_2 defined from $(\mathcal{K}^{\mathbb{T}}, \Delta^{\mathbb{T}})$ as they appear in our previous examples:

$$A_1 = \langle\{A(x,i) \wedge B(x,y,i) \wedge E(x,z,i) \succ\!\!- \neg C(x,i+1)\}, \neg C(x,i+1)\rangle$$

$$A_2 = \langle\{A(x,i) \wedge B(x,y,i) \succ\!\!- C(x,i+1), C(x,i+1) \succ\!\!- D(x,[i+2,i+6])\}, D(x,[i+2,i+6])\rangle$$

Definition 12. Let $\langle A, h\rangle$ be a temporal argument for h, and $\langle S, j\rangle$ a temporal argument for j, such that $S \subseteq A$. We will say that $\langle S, j\rangle$ is a *temporal sub-argument* of $\langle A, h\rangle$, and we will denote it through $\langle S, j\rangle \sqsubseteq \langle A, h\rangle$.

Example 4. $A_2' = \langle\{A(x,i) \wedge B(x,y,i) \succ\!\!- C(x,i+1)\}, C(x,i+1)\rangle \sqsubseteq A_2$.

Definition 13. Let $(\mathcal{K}^{\mathbb{T}}, \Delta^{\mathbb{T}})$ be a temporal defeasible structure of $\mathcal{A}^{\mathbb{T}}$. $\mathbb{T}$**AStruc**$(\Delta^{\mathbb{T}\downarrow})$ will be the set of temporal arguments that can be constructed from $(\mathcal{K}^{\mathbb{T}}, \Delta^{\mathbb{T}\downarrow})$.

Example 5. Let us note as $A^{\downarrow}$ a grounded argument A, then $\mathbb{T}$**AStruc**$(\Delta^{\mathbb{T}\downarrow}) = \{A_1^{\downarrow}, A_2^{\downarrow}\}$:

$$A_1^{\downarrow} = \langle\{A(a,2) \wedge B(a,b,2) \wedge E(a,e,2) \succ\!\!- \neg C(a,3)\}, \neg C(a,3)\rangle$$
$$A_2^{\downarrow} = \langle\{A(a,2) \wedge B(a,b,2) \succ\!\!- C(a,3), C(a,3) \succ\!\!- D(a,[4,8])\}, D(a,[4,8])\rangle$$

Next we consider temporal references that will guide the argumentation process.

Definition 14. Let $\varphi = [\neg]P(\alpha, i)$ or $\varphi = [\neg]P(\alpha, I)$ be a temporal literal, i.e. one with a temporal reference, and Φ a set of temporal literals. The *temporal reference of* φ, $\rho(\varphi)$, is the set of instants mentioned in it. That is to say, $\rho : \Phi \to 2^{|\mathcal{T}|}$ is defined as follows:

$$\rho(\varphi) = \begin{cases} \{i\} & \text{if} \quad \varphi = [\neg]P(\alpha, i) \\ \{i' \in \mathcal{T} | I = [i_1, i_2] \in \mathcal{I}, i_1 \leqslant i' \leqslant i_2,\} & \text{if} \quad \varphi = [\neg]P(\alpha, I) \end{cases}$$

We extend this function to a set of temporal literals L: the *common temporal reference of* L, $\rho(L)$, is the intersection of the temporal references of all $\varphi \in L$, *i.e.*, $\bigcap_{\varphi \in L} \rho(\varphi)$.

Observation: Let h_1 and h_2 be two literals and $\rho(h_1) \cap \rho(h_2) = r$, then $r = \emptyset$ or $r \subseteq \mathcal{T}$ or $r = \{r_1, \ldots, r_n\}$ such that $r_1 < \ldots < r_n$ and $[r_1, r_n] \subseteq \mathcal{I}$.

Definition 15. Given two temporal arguments $\langle A_1, h_1\rangle$ and $\langle A_2, h_2\rangle$, A_1 for h_1 and A_2 for h_2 *are in disagreement at least about an instant i*, $\langle A_1, h_1\rangle \bowtie_{\mathbb{T}} \langle A_2, h_2\rangle$, if and only if $\rho(\{h_1, h_2\}) \neq \emptyset$ and $\mathcal{K}^{\mathbb{T}} \cup \{h_1, h_2\} \vdash \bot$.

A common temporal reference is required between the temporal references of the arguments involved in the conflict. There are several ways to fulfill this requirement. It could happen when both arguments sustain a contradictory thesis like $P(A(c_1,\ldots,c_n),r_1)$ and $\neg P(A(c_1,\ldots,c_n),r_2)$ where (a) r_1 and r_2 are the same instant, (b) r_1 is an interval $[i_1,i_2]$ and r_2 is an instant i such that $i_1 \leqslant i \leqslant i_2$, or (c) r_1 and r_2 are intervals $[i_1,i_2]$ and $[i_3,i_4]$ respectively with some part in common, i.e. $\{i_1,\ldots,i_2\} \cap \{i_3,\ldots,i_4\} \neq \emptyset$.

Definition 16. A temporal argument $\langle A_1,h_1\rangle$ *counter-argues* another temporal argument $\langle A_2,h_2\rangle$ in a basic literal h, if and only if there exists a sub-argument $\langle A,h\rangle$ of $\langle A_2,h_2\rangle$ such that $\langle A_1,h_1\rangle$ and $\langle A,h\rangle$ are in disagreement (in at least an instant i).

In the argumentation process we can naturally get contradictory arguments, and it then becomes necessary to have a mechanism to decide which is preferable.

Definition 17. Let $\succ$ be a partial order defined over elements of $\mathbb{T}$**AStruc**$(\Delta^{\mathbb{T}\downarrow})$. We will say that a temporal argument $\langle A_1,h_1\rangle$ *defeats* another $\langle A_2,h_2\rangle$, $\langle A_1,h_1\rangle \gg_{\mathbf{tdef}} \langle A_2,h_2\rangle$, if and only if there exists a sub-argument $\langle A,h\rangle$ of $\langle A_2,h_2\rangle$ such as $\langle A_1,h_1\rangle$ counter-argues $\langle A_2,h_2\rangle$ in h and $\langle A_1,h_1\rangle \succ \langle A,h\rangle$.

This is another key aspect of every argumentation system since it allows to have an explicit preference criterion among arguments. In consequence, it guides the process of inference allowing, every time that it is possible, to select the most reasonable conclusions in some sense.

Historically, the comparison criterion that has been most widely used for such purposes is that of specificity (Poole, 1985). Intuitively, we can understand specificity as a way of preferring the best-informed arguments. Next, we propose an adaptation to our temporal environment of this criterion by means of the following definition.

Definition 18. Let $\mathscr{D}^{\mathbb{T}} = \{a \in Lit(\mathscr{K}^{\mathbb{T}} \cup \Delta^{\mathbb{T}}) : \mathscr{K}^{\mathbb{T}} \cup \Delta^{\mathbb{T}\downarrow} \mid\!\sim a\}$ where $Lit(\mathscr{K}^{\mathbb{T}} \cup \Delta^{\mathbb{T}})$ is the set of grounded literals built from atoms and predicates of $\mathscr{K}^{\mathscr{T}}$ and $\Delta^{\mathscr{T}}$. Given $\langle A_1,h_1\rangle, \langle A_2,h_2\rangle \in$ $\mathbb{T}$**AStruc**$(\Delta^{\mathbb{T}\downarrow})$, we will say that A_1 for h_1 is *strictly more specific than A_2 for h_2 in an instant i*, denoted $\langle A_1,h_1\rangle \succ_{\mathbf{tspec}} \langle A_2,h_2\rangle$, if and only if:

1. $\rho(\{h_1, h_2\}) \supseteq \{i\}$
2. for every $S \subseteq \mathscr{D}^{\mathbb{T}}$ if $\mathscr{K}^{\mathbb{T}}_G \cup S \cup A_1 \mid\!\sim h_1$
   and $\mathscr{K}^{\mathbb{T}}_G \cup S \not\mid\!\sim h_1$ (S non-trivially activates $\langle A_1,h_1\rangle$)
   then: $\mathscr{K}^{\mathbb{T}}_G \cup S \cup A_2 \mid\!\sim h_2$ (S activates $\langle A_2,h_2\rangle$)
3. there exists $S \subseteq \mathscr{D}^{\mathbb{T}}$ such that: $\mathscr{K}^{\mathbb{T}}_G \cup S \cup A_2 \mid\!\sim h_2$
   and $\mathscr{K}^{\mathbb{T}}_G \cup S \not\mid\!\sim h_2$ (S non-trivially activates $\langle A_2,h_2\rangle$)
   and $\mathscr{K}^{\mathbb{T}}_G \cup S \cup A_1 \not\mid\!\sim h_1$ (S does not activate $\langle A_1,h_1\rangle$)

Example 6. If we consider the following sub-argument of $A_2^{\downarrow}$:

$$A_2'^{\downarrow} = \langle\{A(a,2) \wedge B(a,b,2) \succ\!\!- C(a,3)\}, C(a,3)\rangle$$

and argument $A_1^{\downarrow}$ then $\rho(\{\neg C(a,3), C(a,3)\}) = \{3\}$ and $A_1^{\downarrow} \bowtie_{\mathbb{T}} A_2'^{\downarrow}$ in 3. It can be seen that $A_1^{\downarrow} \succ_{\mathbf{tspec}} A_2'^{\downarrow}$, so $A_1^{\downarrow} \gg_{\mathbf{tdef}} A_2'^{\downarrow}$ and as a consequence $A_1^{\downarrow} \gg_{\mathbf{tdef}} A_2^{\downarrow}$.

Since the specificity criterion is based on the structure of the arguments it has the advantage of being independent from the application domain. Although we have adapted this criterion for our temporal environment, we have separated it from the system because sometimes it could be desired to replace it or combine it with another criterion. For example, since information in our knowledge base is associated to time, we could desire to have it in mind when deciding about the quality of two arguments. It is also possible to use domain-dependent criteria or a combination of strategies to decide which argument is better. Now we will proceed to define when an argument is considered a justification of a thesis through the notion of 'supporting' and 'interfering' arguments (Pollock, 1987).

Definition 19. The temporal arguments are active in two modalities, supporting or interfering arguments (*S-arguments* and *I-arguments* respectively):

1. every argument is an S-argument and I-argument (of level 0);
2. an argument $\langle A_1, h_1 \rangle$ is an S-argument (of level $(n+1)$) if and only if there is not an I-argument of level n $\langle A_2, h_2 \rangle$ such that for an h, $\langle A_2, h_2 \rangle$ counter-argue $\langle A_1, h_1 \rangle$ in h;
3. an argument $\langle A_1, h_1 \rangle$ is an I-argument (of level $(n+1)$) if and only if there is not an I-argument of level n $\langle A_2, h_2 \rangle$ such that $\langle A_2, h_2 \rangle$ defeats $\langle A_1, h_1 \rangle$.

Finally we define when a thesis can be said to be justified by an argument:

Definition 20. We will say that a temporal argument $\langle A, h \rangle$ *justifies* h if and only if there exists m such that, for every $n \geqslant m$, $\langle A, h \rangle$ is an S-argument of level n for h.

Example 7. $A_1^{\downarrow}$ is a justification for $\neg C(a, 3)$ and this prevents $D(a, [4, 8])$ from being inferred. It also allows $A_1^{\downarrow}$ to be, indirectly, a justification for $\neg D(a, [4, 6])$. The interval $[4, 6]$ is the common temporal reference obtained as: $\rho(\{D(a, [4, 8]), \neg D(a, [4, 6])\}) = \{4, 5, 6\}$. It must be observed that rules in $\mathscr{K}_G^{\mathbb{T}}$ are not included in arguments.

This means that a thesis will be said to be justified by an argument when the argument supporting the conclusion is better, under the preference criteria, than its counter-arguments. In the following section we consider some computational aspects of the interaction between the argumentation and temporal layers.

4. Persistency

We consider some axioms to represent the assumption that properties tend to keep their truth values throughout time. The idea is quite similar to *explanation closure axioms* as proposed in Schubert (1990). This strategy is similar to frame axioms but with nicer properties regarding computability (see Schubert, 1990, for an analysis of its advantages). Other contemporary research in this topic the reader will find of interest is Sandewall (1994) and Shanahan (1997). These are semantic-based approaches based on minimization of models, while explanation closure technique is mainly a syntactic one.

The closure axioms technique specifies for each property which events could change it. Only if we have reason to conclude that some of those events with the capability to change the property have occurred does it make sense to assume the property has changed.

Similarly, we consider axioms saying when change has occurred and defeasible rules making explicit that we assume a property has not changed if we are unable to find a justification for the thesis that it has changed. Since we can code this kind of rule in the argumentation system then we have a finite and very low number of them. Also our approach leads to a more direct and natural way to use persistency as a difference from the approach given in Allen and Ferguson (1994), where persistency is proved by using contrapositive in a 'proof by contradiction' strategy. We will use in this section predicates $\text{Change}_{at}^{+\,-}(p, i))$ and $\text{Change}_{in}^{+\,-}(p, I))$ to indicate that a proposition p changes its truth value from being true to false at an instant i or in an interval I respectively. The following axioms allow the detection of these situations:

$$\forall_{\mathcal{P}}\ p\ \forall_{\mathcal{T}} i(\text{Holds}_{at}(p, i-1) \wedge \neg\text{Holds}_{at}(p, i) \rightarrow \text{Change}_{at}^{+\,-}(p, i)) \tag{24}$$

$$\forall_{\mathcal{P}}\ p\ \forall_{\mathcal{I}} I, I'(\text{MEETS}(I, I') \\ \wedge\text{Holds}_{on}(p, I) \wedge \neg\text{Holds}_{on}(p, I') \rightarrow \text{Change}_{in}^{+\,-}(p, I') \tag{25}$$

We can also consider analogous axioms for $\text{Change}_{at}^{-\,+}$ and $\text{Change}_{in}^{-\,+}$ for properties changing from being false to being true. The general form of axioms to prove persistency can be briefly described as: 'if a property p is true (false) and we cannot justify reasons to believe it has changed then we can assume it will remain true for some time'. We will use $\mathsf{not}\ A(x_1, \ldots, x_n)$ to represent the impossibility to justify $A(x_1, \ldots, x_n)$. not is a meta-symbol that can only be applied to positive literals. The reader can find a formalization for not in García and Simari (1999).

We do not ask for specific rules stating when a property changes its value but instead we assume a set of rules as in the closure axiom technique stating which properties change as a consequence of other events and actions:

$$\forall_{\mathcal{P}} p \forall_{\mathcal{T}} i(\text{Holds}_{at}(p, i) \wedge \mathsf{not}\text{Change}_{at}^{+-}(p, i+1) \succ\!\!- \text{Holds}_{at}(p, i+1)) \tag{26}$$

$$\forall_{\mathcal{P}} p \forall_{\mathcal{I}} I, I'(\text{Holds}_{on}(p, I) \\ \wedge\text{MEETS}(I, I') \wedge \mathsf{not}\text{Change}_{in}^{+-}(p, I')) \succ\!\!- \text{Holds}_{on}(p, I') \tag{27}$$

Using these axioms we can get the following results giving a formalization for the assumption that we can suppose that 'a property does not change unless we have some reason to believe it did'.

Lemma 1.

(a) $\forall_{\mathcal{P}}\ p\ \forall_{\mathcal{I}}\ I(\mathsf{not}\text{Change}_{in}^{+-}(p, I) \rightarrow$

$$(\forall_{\mathcal{T}}\ i(begin(I) \leqslant i \leqslant end(I) \rightarrow \mathsf{not}\text{Change}_{at}^{+-}(p, i))))$$

(b) $\forall_{\mathcal{P}}\ p\ \forall_{\mathcal{I}}\ I, I'\ (\mathsf{not}\text{Change}_{in}^{+-}(p, I) \wedge \mathsf{not}\text{Change}_{in}^{+-}(p, I') \wedge \text{MEETS}(I, I')) \rightarrow$

$$\mathsf{not}\text{Change}_{in}^{+-}(p, [begin(I), end(I')]))$$

Proof. The first implication follows from the definitions of interval and not while the second follows from the definitions of MEETS and not. □

Theorem 1.
$\forall_{\mathscr{P}}\ p\ \forall_{\mathscr{T}}\ i, i'\ (\text{Holds}_{at}(p,i) \wedge \mathsf{notChange}_{in}^{+-}(p,[i+1,i'])\ {\succ\!\!-}\ \text{Holds}_{at}(p,i'))$

$$\forall_{\mathscr{P}} p \forall_{\mathscr{I}} I, I'(\text{Holds}_{on}(p,I) \wedge end(I) \leqslant begin(I') \wedge \mathsf{notChange}_{in}^{+-}(p,[end(I)+1, end(I')]) \ {\succ\!\!-}\ \text{Holds}_{on}(p,I')$$

Proof. The first result follows from axiom 26 and lemma 1 (a) while the later follows from axiom 27 and lemma 1 (b). □

We will also use another preference criterion related to persistency giving priority to knowledge-based arguments instead of those belief-based arguments like those based on persistency.

Definition 21. Let $\langle A_1, h_1\rangle, \langle A_2, h_2\rangle \in$ **TAStruc**$(\Delta^{\mathscr{T}\downarrow})$, we say that A_1 for h_1 is *preferred under persistency* to A_2 for h_2, noted $\langle A_1, h_1\rangle \succ_{\mathbf{tpers}} \langle A_2, h_2\rangle$, if and only if $\langle A_2, h_2\rangle$ use persistency and $\langle A_1, h_1\rangle$ does not.

In the next sections we assume the following precedence order (Prakken, 1993) between the preference criterion: $\Re = \{\succ_{\mathbf{tspec}}, \succ_{\mathbf{tpers}}\}, \succ_{\mathbf{tspec}} >_{\Re} \succ_{\mathbf{tpers}}$. This means we always try to apply specificity first. When the arguments are incomparable under specificity or they are equi-specific we apply the persistency criteria.

5. Some Examples

We illustrate the behavior of the system showing how to formalize some well-known problems of the literature. The first couple of problems where proposed in Sandewall (1994), while the others where considered in Ferguson (1995) in relation to the TRAINS project. We will use interval-based temporal references to make the comparison between our solutions for these problems and those proposed in Ferguson (1995) easier.

We will also show arguments in their tree-form (García et al., 1993). In the tree-form of an argument for h such as $\langle\{p_1,\ldots,p_n\},h\rangle$ the root will be h and its sons will be $p_1,\ldots,p_n$. Each p_i in turn could be a fact or a conclusion of another argument. In the former case it will become a leaf in the tree. In the second case it will be the root of another tree. The notation we shall use in the trees is a bit different to that used in the arguments. This slight difference helps to keep trees smaller. For example, $A@i$ means $P(A,i)$ and $A@I$ means $P(A,I)$ where P could be understood from the meaning of A or a previous formal description of the argument represented in the tree. Also we use TC as a way to shorten *Temporal Constraints*. These constraints are those mentioned in a previous formal description of the argument being represented. We use the notation and conventions defined in $\mathscr{L}^{\mathbb{T}}$ to represent temporal concepts in the argumentation level.

5.1. Hidden Turkey Shoot Problem

This is a variation proposed in Sandewall (1994) of the widely known Yale shooting problem. The scenario considers a turkey that will be shot and the possibility that if it is able to hear the gun being loaded then it can hide and save its life. Otherwise, shooting the gun will cause its death.

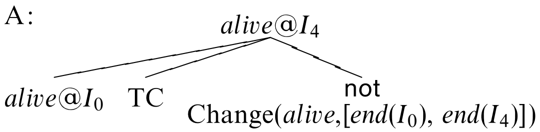


Fig. 1. Argument tree for $alive@I_4$.

$$\text{MEETS}(I_0,I_1) \wedge \text{MEETS}(I_1,I_2) \wedge \text{MEETS}(I_2,I_3) \wedge \text{MEETS}(I_3,I_4)$$
$$\text{Holds}_{on}(alive,I_0) \wedge \neg\text{Holds}_{on}(loaded,I_0) \wedge \neg\text{Holds}_{on}(hidden,I_0)$$
$$\text{Do}_{on}(loading,I_1) \wedge \text{Do}_{on}(shooting,I_3)$$
$$(HT1)\ \forall_{\mathcal{I}} I\ (\text{Do}_{on}(loading,I) \succ\!\!- \text{Occurs}_{on}(load,I))$$
$$(HT2)\ \forall_{\mathcal{I}} I\ (\text{Do}_{on}(shooting,I) \wedge \text{Holds}_{on}(loaded,I) \succ\!\!- \text{Occurs}_{on}(shoot,I))$$
$$(HT3)\ \forall_{\mathcal{I}} I,I'(\text{Occurs}_{on}(load,I) \succ\!\!- \text{Holds}_{on}(loaded,I') \wedge \text{MEETS}(I,I'))$$
$$(HT4)\ \forall_{\mathcal{I}} I,I'(\text{Occurs}_{on}(load,I) \wedge \neg\text{Holds}_{on}(deaf,I) \succ\!\!- \text{Occurs}_{on}(hide,I))$$
$$(HT5)\ \forall_{\mathcal{I}} I,I'(\text{Occurs}_{on}(hide,I) \succ\!\!- \text{Holds}_{on}(hidden,I') \wedge \text{MEETS}(I,I'))$$
$$(HT6)\ \forall_{\mathcal{I}} I,I'(\text{Occurs}_{on}(shoot,I) \wedge \text{Holds}_{on}(loaded,I) \succ\!\!- \neg\text{Holds}_{on}(loaded,I') \wedge \text{MEETS}(I,I'))$$
$$(HT7)\ \forall_{\mathcal{I}} I,I'(\text{Occurs}_{on}(shoot,I) \wedge \text{Holds}_{on}(alive,I) \wedge \neg\text{Holds}_{on}(hidden,I) \succ\!\!- \neg\text{Holds}_{on}(alive,I') \wedge \text{MEETS}(I,I'))$$
$$(HT8)\ \forall_{\mathcal{A}} a \forall_{\mathcal{I}} I,I_1,I_3(\text{Do}_{on}(a,I) \leftrightarrow ((a = loading \wedge I = I_1) \vee (a = shooting \wedge I = I_3)))$$

If we assume $\neg\text{Holds}_{on}(deaf,I_1)$ then an argument can be built to support that $\text{Holds}_{on}(alive,I_4)$ based on the tendency to persist of the property 'alive'. From $\text{Do}_{on}(loading,I_1)$ and $(HT1)$ we obtain $\text{Occurs}_{on}(load,I_1)$. By the hypothesis and $(HT4)$ we get $\text{Occurs}_{on}(hide,I_1)$. Using $(HT5)$ we can get $\text{Holds}_{on}(hidden,I_2)$. Using the persistency Theorem 1, we have $\text{Holds}_{on}(hidden,I_3)$. As there is no way to show $\neg\text{Holds}_{on}(alive,I_4)$, by the persistency Theorem 1, $\text{Holds}_{on}(alive,I_4)$ follows from $\text{Holds}_{on}(alive,I_0)$. The above-described reasoning is summarized in the following argument:

$$A = \langle\{\text{Holds}_{on}(alive,I_0) \wedge (end(I_0) \leqslant begin(I_4)) \wedge \text{notChange}^{+-}_{in}(alive,[end(I_0),end(I_4)]) \succ\!\!- \text{Holds}_{on}(alive,I_4)\}, \text{Holds}_{on}(alive,I_4)\rangle$$

which is represented in the tree of Fig. 1.

If we instead assume $\text{Holds}_{on}(deaf,I_1)$, by persistency axiom 27, we get $\neg\text{Holds}_{on}(hidden,I_3)$ and as a consequence $\neg\text{Holds}_{on}(alive,I_4)$. By using $\text{Do}_{on}(loading,I_1)$, $(HT1)$ and $(HT3)$ we get $\text{Holds}_{on}(loaded,I_2)$ as before and by persistency, through Axiom 27, we can get that $\text{Holds}_{on}(loaded,I_3)$. This fact together with $\text{Do}_{on}(shooting,I_3)$ can be used with $(HT2)$ to infer $\text{Occurs}_{on}(shoot,I_3)$. By $(HT7)$ we obtain $\neg\text{Holds}_{on}(alive,I_4)$. Argument B can be defined in a modular way to represent the previous rationale in support of $\neg\text{Holds}_{on}(alive,I_4)$:

$$B_1 = \langle\{\text{Do}_{on}(loading,I_1) \succ\!\!- \text{Occurs}_{on}(load,I_1), \text{Occurs}_{on}(load,I_1) \succ\!\!- \text{Holds}_{on}(loaded,I_2)\}, \text{Holds}_{on}(loaded,I_2)\rangle$$

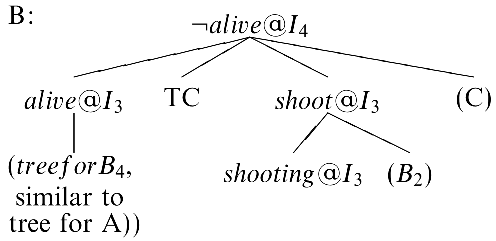


Fig. 2. Argument tree for $\neg alive@I_4$.

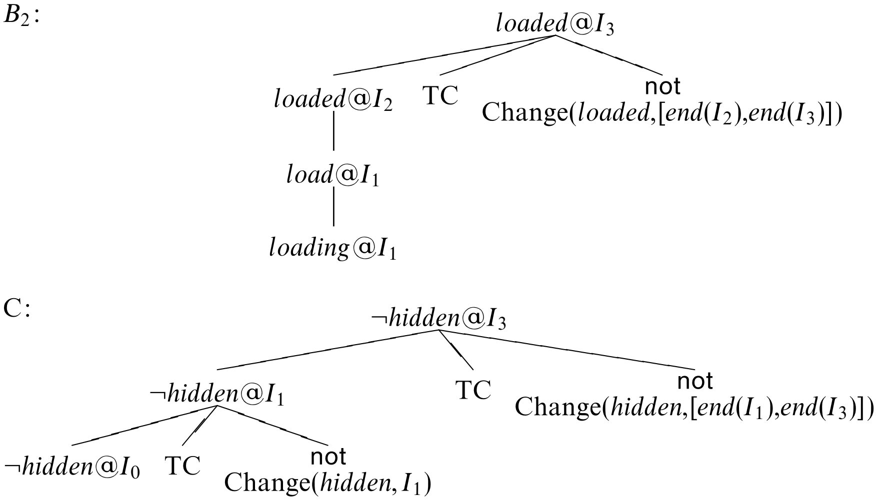


Fig. 3. Argument trees for $loaded@I_3$ and $\neg hidden@I_3$.

$$B_2 = \langle\{\text{Holds}_{on}(loaded, I_2) \wedge \text{MEETS}(I_2, I_3) \wedge \text{notChange}_{in}^{+-}(loaded, I_3) \succ\!\!- \text{Holds}_{on}(loaded, I_3)\}, \text{Holds}_{on}(loaded, I_3)\rangle$$

$$B_3 = \langle\{\text{Do}_{on}(shooting, I_3) \wedge \text{Holds}_{on}(loaded, I_3) \succ\!\!- \text{Occurs}_{on}(shoot, I_3)\}, \text{Occurs}_{on}(shoot, I_3)\rangle$$

$$B_4 = \langle\{\text{Holds}_{on}(alive, I_0) \wedge (end(I_0) \leqslant begin(I_3)) \wedge \text{notChange}_{in}^{+-}(alive, [end(I_0), end(I_3)]) \succ\!\!- \text{Holds}_{on}(alive, I_3)\}, \text{Holds}_{on}(alive, I_3)\rangle$$

$$D = \langle\{\neg\text{Holds}_{on}(hidden, I_0) \wedge \text{MEETS}(I_0, I_1) \wedge \text{notChange}_{in}^{-+}(hidden, I_1) \succ\!\!- \neg\text{Holds}_{on}(hidden, I_1)\}, \neg\text{Holds}_{on}(hidden, I_1)\rangle$$

$$C = \langle\{\neg\text{Holds}_{on}(hidden, I_1) \wedge (end(I_1) \leqslant begin(I_3)) \wedge \text{notChange}_{in}^{-+}(hidden, [end(I_1), end(I_3)]) \succ\!\!- \neg\text{Holds}_{on}(hidden, I_3)\}, \neg\text{Holds}_{on}(hidden, I_3)\rangle$$

$$B = \langle\{\text{Holds}_{on}(alive, I_3) \wedge \text{Occurs}_{on}(shoot, I_3) \wedge \text{MEETS}(I_3, I_4) \wedge \neg\text{Holds}_{on}(hidden, I_3)\} \succ\!\!- \neg\text{Holds}_{on}(alive, I_4)\}, \neg\text{Holds}_{on}(alive, I_4)\rangle$$

These arguments are shown as trees in Figs 2 and 3.

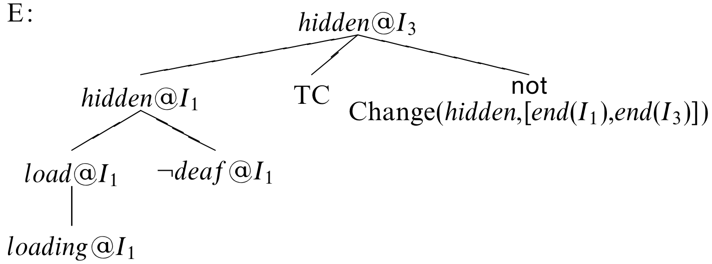


Fig. 4. Argument tree for $hidden@I_3$.

If $\text{Holds}_{on}(deaf, I_1)$ then $B \bowtie_{\mathbb{T}} A$, $B \succ_{\text{tspec}} A$ and $B \gg_{\text{tdef}} A$. But, if $\neg\text{Holds}_{on}(deaf, I_1)$ we have:

$$F = \langle \{ \text{Do}_{on}(loading, I_1) \succ\!\!- \text{Occurs}_{on}(load, I_1), \; \text{Occurs}_{on}(load, I_1) \wedge \neg\text{Holds}_{on}(deaf, I_1) \succ\!\!- \text{Holds}_{on}(hidden, I_1)\}, \; \text{Holds}_{on}(hidden, I_1) \rangle$$

$$E = \langle \{ \text{Holds}_{on}(hidden, I_1) \wedge (end(I_1) \leqslant begin(I_3)) \wedge \; \text{notChange}_{in}^{+-}(hidden, [end(I_1), end(I_3)]) \succ\!\!- \text{Holds}_{on}(hidden, I_3)\}, \; \text{Holds}_{on}(hidden, I_3) \rangle$$

then $F \bowtie_{\mathbb{T}} D$. The argument E is represented in the tree of Fig. 4.

We cannot use specificity with F and D because they are incomparable under this criterion so the persistency-based criterion is applied and we prefer arguments not using this notion as the only reason to support its conclusion. The task to compare arguments using the notion of persistency could be simplified considering that

$$\forall_{\mathscr{P}}\, p \; (\text{Holds}_{on}(p, I_1) \wedge (end(I_1) \leqslant begin(I_n)) \wedge \text{notChange}_{in}^{+-}(p, [end(I_1), end(I_n)]) \succ\!\!- \text{Holds}_{on}(p, I_n))$$

could be rewritten as:

$$\forall_{\mathscr{P}} p(\text{Holds}_{on}(p, I_1) \wedge \text{Holds}_{on}(p, I_2) \wedge \ldots \wedge \text{Holds}_{on}(p, I_n) \succ\!\!- \text{Holds}_{on}(p, I_n))$$

Under this criterion F is preferred so $F \succ_{\text{tpers}} D$ and $F \gg_{\text{tdef}} D$. As a conclusion we have that $E \bowtie_{\mathbb{T}} C$, $E \succ_{\text{tpers}} C$ and $E \gg_{\text{tdef}} C$. As a result, when $\neg\text{Holds}_{on}(deaf, I_1)$ is assumed E acts as an *undercutting defeater* (Prakken, 1993) of B so A is the preferred argument.

5.2. Russian Turkey Shoot Problem

This scenario (Sandewall, 1994) introduces the notion of uncertainty on events. Uncertainty is provided by spinning the chamber of the gun. As a result of this action the bullet may or may not be ready for firing, leading to two very different results. To make the comparison between Ferguson and Allen's proposal and this one easier, the presentation of the problem employed in Allen and Ferguson (1994) will be adopted:

$\text{MEETS}(I_0, I_1) \wedge \text{MEETS}(I_1, I_2) \wedge \text{MEETS}(I_2, I_3) \wedge \text{MEETS}(I_3, I_4)$

$\text{Holds}_{on}(alive, I_0) \wedge \neg\text{Holds}_{on}(loaded, I_0)$

$\text{Do}_{on}(loading, I_1) \wedge \text{Do}_{on}(spinning, I_2) \wedge \text{Do}_{on}(shooting, I_3)$
$(R1)\ \forall_{\mathscr{I}} I\ (\text{Do}_{on}(loading, I) \succ\!\!- \text{Occurs}_{on}(load, I))$
$(R2)\ \forall_{\mathscr{I}} I\ (\text{Do}_{on}(shooting, I) \wedge \text{Holds}_{on}(loaded, I) \succ\!\!- \text{Occurs}_{on}(shoot, I))$
$(R3)\ \forall_{\mathscr{I}} I\ (\text{Do}_{on}(spin, I) \succ\!\!- \text{Occurs}_{on}(spin, I))$
$(R4)\ \forall_{\mathscr{I}} I, I'(\text{Occurs}_{on}(load, I) \succ\!\!- \text{Holds}_{on}(loaded, I') \wedge\ \text{MEETS}(I, I'))$

$$(R5)\ \forall_{\mathscr{I}} I, I'(\text{Occurs}_{on}(shoot, I) \wedge \text{Holds}_{on}(loaded, I) \succ\!\!- \neg\text{Holds}_{on}(loaded, I') \wedge\ \text{MEETS}(I, I'))$$

$$(R6)\ \forall_{\mathscr{I}} I, I'(\text{Occurs}_{on}(shoot, I) \wedge \text{Holds}_{on}(alive, I) \succ\!\!- \neg\text{Holds}_{on}(alive, I') \wedge\ \text{MEETS}(I, I'))$$

$$(R7)\ \forall_{\mathscr{I}} I, I'\ (\text{Occurs}_{on}(spin, I) \succ\!\!- ((\text{Change}^{+-}_{in}(loaded, I') \vee \text{Holds}_{on}(loaded, I')) \wedge\ \text{MEETS}(I, I'))$$

$$(R8)\ \forall_{\mathscr{I}} I, I'\ (\text{Holds}_{on}(loaded, I) \wedge \text{Change}^{+-}_{in}(loaded, I) \succ\!\!- \neg\text{Holds}_{on}(loaded, I') \wedge\ \text{MEETS}(I, I'))$$

$$(R9)\ \forall_{\mathscr{A}} a \forall_{\mathscr{I}} I, I_1, I_2, I_3(\text{Do}_{on}(a, I) \leftrightarrow ((a = loading \wedge I = I_1) \vee (a = spinning \wedge I = I_2) \vee (a = shooting \wedge I = I_3)))$$

As with the previous example the argument A can be built based on the persistency of the property 'alive'. We omit the tree for this argument as it is the same as that of Fig. 1.

$$A = \langle\{\text{Holds}_{on}(alive, I_0) \wedge (end(I_0) \leqslant begin(I_4)) \wedge \mathsf{notChange}^{+-}_{in}(alive, [end(I_0), end(I_4)]) \succ\!\!- \text{Holds}_{on}(alive, I_4)\}, \text{Holds}_{on}(alive, I_4)\rangle$$

But there are also two ways to support the conclusion that things happened in the other way around. One way arises by considering that the gun remains loaded after I_1 because of the persistency axiom. The shoot can be done in I_3 getting $\neg\text{Holds}_{on}(alive, I_4)$ using $(R2)$ and $(R6)$. Using this knowledge we can build the following arguments:

$$B_1 = \langle\{\text{Do}_{on}(loading, I_1) \succ\!\!- \text{Occurs}_{on}(load, I_1), \text{Occurs}_{on}(load, I_1) \succ\!\!- \text{Holds}_{on}(loaded, I_2)\}, \text{Holds}_{on}(loaded, I_2)\rangle$$

$$B_2 = \langle\{\text{Holds}_{on}(loaded, I_2) \wedge (end(I_0) \leqslant begin(I_3)) \wedge \mathsf{notChange}^{+-}_{in}(loaded, [end(I_0), end(I_3)]) \succ\!\!- \text{Holds}_{on}(loaded, I_3)\}, \text{Holds}_{on}(loaded, I_3)\rangle$$

$$B_3 = \langle\{\text{Do}_{on}(shooting, I_3) \wedge \text{Holds}_{on}(loaded, I_3) \succ\!\!- \text{Occurs}_{on}(shoot, I_3)\}, \text{Occurs}_{on}(shoot, I_3)\rangle$$

$$B_4 = \langle\{\text{Holds}_{on}(alive, I_0) \wedge (end(I_0) \leqslant begin(I_3)) \wedge \mathsf{notChange}^{+-}_{in}(alive, [end(I_0), end(I_3)]) \succ\!\!- \text{Holds}_{on}(alive, I_3)\}, \text{Holds}_{on}(alive, I_3)\rangle$$

We can then use B_1, B_2, B_3 and B_4 to build

$$B = \langle\{\text{Holds}_{on}(alive, I_3) \wedge \text{Occurs}_{on}(shoot, I_3) \wedge\ \text{MEETS}(I_3, I_4) \succ\!\!- \neg\text{Holds}_{on}(alive, I_4)\}, \neg\text{Holds}_{on}(alive, I_4)\rangle$$

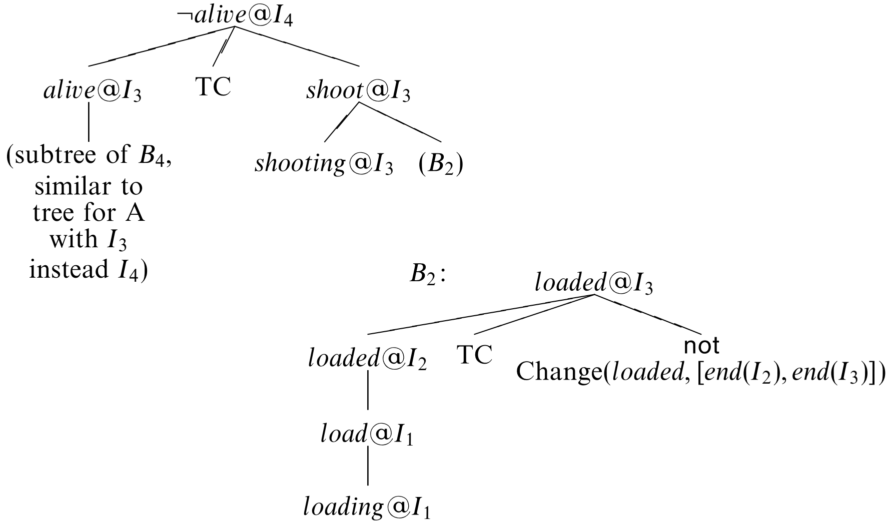


Fig. 5. Tree of the argument for $\neg alive@I_4$ (first case).

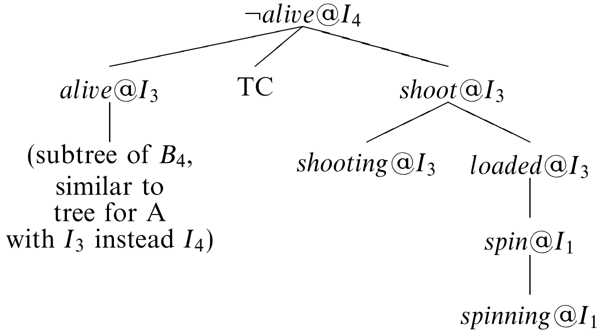


Fig. 6. Tree of the argument for $\neg alive@I_4$ (second case).

such that $B \bowtie_{\mathbb{T}} A$, $B \succ_{\text{tspec}} A$ and $B \gg_{\text{tdef}} A$. Arguments B_1 and B_2 can be replaced if we use the conclusion $\text{Holds}_{on}(loaded, I') \wedge \text{MEETS}(I, I')$ of ($R7$) to get the argument

$$C = \langle\{\text{Do}_{on}(spinning, I_2) \succ\!\!- \text{Occurs}_{on}(spin, I_2), \text{Occurs}_{on}(spin, I_2) \succ\!\!- \text{Holds}_{on}(loaded, I_3)\}, \text{Holds}_{on}(loaded, I_3)\rangle$$

Arguments C, B_3 and B_4 also allow to support B as above such that $B \bowtie_{\mathbb{T}} A$, $B \succ_{\text{tspec}} A$ and $B \gg_{\text{tdef}} A$. Arguments B_1, B_2, B_3, B_4 and B are shown as a tree in Fig. 5. The line of reasoning sustained by arguments C, B_3, B_4 and B is shown in Fig. 6.

There are then two ways to conclude $\neg\text{Holds}_{on}(alive, I_4)$ which in turn corresponds to each possible way of loading the gun, i.e., by putting in a bullet or by

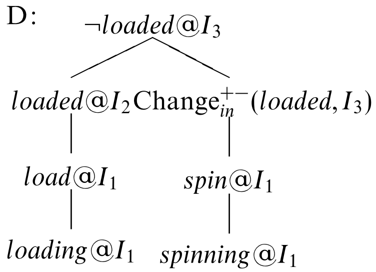


Fig. 7. Tree of the argument for $\neg loaded@I_3$.

spinning the chamber. Both ways represent more specific knowledge than that used to build argument A. If we instead consider

$$\text{Occurs}_{on}(spin, I) \succ\!\!- \text{Change}^{+-}_{in}(loaded, I') \wedge \text{MEETS}(I, I')$$

spinning the chamber of the gun stops the persistence of 'loaded' and the shooting cannot be done.

The argument in this case is as follows (see Fig. 7 for a tree-like representation):

$$\begin{aligned} D_2 = \langle\{&\text{Do}_{on}(spinning, I_2) \succ\!\!- \text{Occurs}_{on}(spin, I_2),\\ &\text{Occurs}_{on}(spin, I_2) \succ\!\!- \text{Change}^{+-}_{in}(loaded, I_3)\},\\ &\text{Change}^{+-}_{in}(loaded, I_3)\rangle \end{aligned}$$

$$\begin{aligned} D = \langle\{&\text{Holds}_{on}(loaded, I_2) \wedge \text{Change}^{+-}_{in}(loaded, I_3) \succ\!\!- \neg\text{Holds}_{on}(loaded, I_3)\},\\ &\neg\text{Holds}_{on}(loaded, I_3)\rangle \end{aligned}$$

Then $\text{notChange}^{+-}_{in}(loaded, I_3)$ can no longer be justified because there is an argument for $\text{Change}^{+-}_{in}(loaded, I_3)$ through D. This forbids the construction of argument B reinstating argument A. As expected, the result of the spinning decides the conclusion.

5.3. Mutually Exclusive Actions

This scenario was proposed in Allen and Ferguson (1994) and deals with actions that cannot be performed together. We simplified the problem assuming this does not change the essential issues addressed. The scenario involves two engines, $n1$ and $n2$, moving between two cities. Engine $n1$ needs to move from Dansville to Corning while $n2$ tries to move in the opposite direction. Just one engine is allowed on each track so the problem arises when an engine needs to use the track while another is still using it. The variable n will represent an engine while c_i, for $i = 1, 2$, will be used to refer to cities. $\text{Do}_{on}(move(n, c_1, c_2, x)$ represents the action of moving with engine n from city c_1 to city c_2 by track x. $\text{Occurs}_{on}(movebetween(n, c_1, c_2, x)$ represents the resulting event.

$$I \lhd\!\rhd I' =_{def} \text{BEFORE}(I, I') \vee \text{MEETS}(I, I') \vee \text{BEFORE}(I', I) \vee \text{MEETS}(I', I)$$

$\text{MEETS}(I_0, I_1) \wedge \text{MEETS}(I_1, I_2)$
$\text{Engine}(n1) \wedge \text{Engine}(n2)$
$\text{Holds}_{on}(at(n1, dansville), I_0) \wedge \text{Holds}_{on}(at(n2, corning), I_0)$
$\text{Do}(move(n1, dansville, corning, track1), I_1)$

$\text{Do}(move(n2, corning, dansville, track2), I_1)$

$(ME1)\ \forall_{\mathscr{W}} n, c_1, c_2, x \forall_{\mathscr{I}} I(\text{Do}(move(n, c_1, c_2, x), I) \succ\!\!- \text{Holds}_{on}(on(n, x), I))$

$$(ME2)\ \forall_{\mathscr{W}} n, c_1, c_2, x \forall_{\mathscr{I}} I(\text{Do}_{on}(move(n, c_1, c_2, x), I) \wedge \text{Holds}_{on}(at(n, c_1), I) \wedge \text{Holds}_{on}(trackclearfor(n, x), I) \succ\!\!- \text{Occurs}_{on}(movebetween(n, c_1, c_2, x), I))$$

$$(ME3)\ \forall_{\mathscr{W}} n, c_1, c_2, x \forall_{\mathscr{I}} I, I'(\text{Occurs}_{on}(movebetween(n, c_1, c_2, x), I)) \succ\!\!- \text{Holds}_{on}(at(n, c_2), I') \wedge \text{MEETS}(I, I'))$$

$$(ME4)\ \forall_{\mathscr{W}} n, x, y \forall_{\mathscr{I}} I, I'(\text{Holds}_{on}(on(n, x), I) \wedge \text{Holds}_{on}(on(n, y), I') \wedge x \neq y \succ\!\!- I \lhd\!\rhd I'$$

$$(ME5)\ \forall_{\mathscr{W}} n, x \forall_{\mathscr{I}} I, I'(\text{Holds}_{on}(trackclearfor(n, x), I) \leftrightarrow \forall n', I'(I' \sqsubseteq I \wedge \text{Engine}(n') \rightarrow \neg\text{Holds}_{on}(on(n', x), I') \vee n' = n))$$

$$(ME6)\ \forall_{\mathscr{W}} n \text{Engine}(n) \leftrightarrow (n = n1) \vee (n = n2)$$

$$(ME7)\ \forall_{\mathscr{A}} a \forall_{\mathscr{I}} I(\text{Do}_{on}(a, I) \leftrightarrow ((a = move(n1, dansville, corning, track1) \wedge I = I_1) \vee (a = move(n2, corning, dansville, track2) \wedge I = I_1)))$$

If an engine has a clear track it then arrives at its destination.

$$A_1 = \langle\{ \text{Holds}_{on}(at(n1, dansville), I_0) \wedge \text{MEETS}(I_0, I_1) \wedge \mathsf{notChange}_{in}^{+-}(at(n1, dansville), I_1) \succ\!\!- \text{Holds}_{on}(at(n1, dansville), I_1)\}, \text{Holds}_{on}(at(n1, dansville), I_1)\rangle$$

$$A_2 = \langle\{\neg\text{Holds}_{on}(on(n2, track1), I_1) \succ\!\!- \text{Holds}_{on}(trackclearfor(n1, track1), I_1)\}, \text{Holds}_{on}(trackclearfor(n1, track1), I_1)\rangle$$

$$A_3 = \langle\{\text{Do}_{on}(move(n1, dansville, corning, track1), I_1) \wedge \text{Holds}_{on}(at(n1, dansville), I_1) \wedge \text{Holds}_{on}(trackclearfor(n1, track1), I_1) \succ\!\!- \text{Occurs}_{on}(movebetween(n1, dansville, corning, track1), I_1)\}, \text{Occurs}_{on}(movebetween(n1, dansville, corning, track1), I_1)\rangle$$

$$A = \langle\{\text{Occurs}_{on}(movebetween(n1, dansville, corning, track1), I_1) \succ\!\!- \text{Holds}_{on}(at(n1, corning), I_2)\}, \text{Holds}_{on}(at(n1, corning), I_2)\rangle$$

The corresponding arguments are represented in the tree of Fig. 8. Because of space restrictions we shall shorten *move between* to ‘mb’, *track clear for* as ‘tcf’, while constants *dansville* and *corning* will be represented by d and c respectively.

If an engine is using one track, e.g. track 1, it is no longer available for other engines and then the engine going from Dansville to Corning cannot use it.

An argument for $\text{Holds}_{on}(on(n2, track1), I_1)$ is a *rebutting defeater* (Prakken, 1993) for A_2 and an *undercutting defeater* (Prakken, 1993) for A:

$$B = \langle\{\text{Do}_{on}(move(n2, corning, dansville, track1), I_1) \wedge \text{Holds}_{on}(trackclearfor(n2, track1), I_1) \succ\!\!- \text{Holds}_{on}(on(n2, track1), I_1)\}, \text{Holds}_{on}(on(n2, track1), I_1)\rangle$$

If we consider the trivial argument $A'_2 = \langle\{\}, \neg\text{Holds}_{on}(on(n2, track1), I_1)\rangle$ we have $B \bowtie_{\mathbb{T}} A'_2$ and $B \succ_{\text{tspec}} A'_2$ as a particular case of specificity and then $B \gg_{\text{tdef}} A'_2$. The corresponding tree-like argument is shown in Fig. 9.

Thus, as expected, one engine in a track forbids the other from using it.

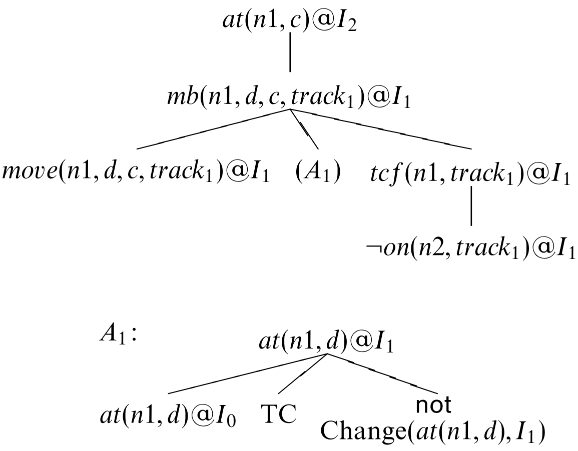


Fig. 8. Argument tree for $at(n1, c)@I_2$.

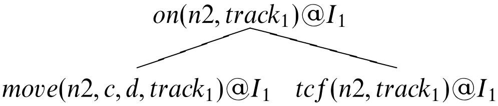


Fig. 9. Argument tree for $on(n2, track_1)@I_1$.

5.4. Synergistic Effects

This problem deals with the possibility and necessity of combining effects of actions to achieve a unique goal. The scenario described in Allen and Ferguson (1994) includes the attempt to decouple a car by activating the decoupler while the engine is moving forward. Again, we assume the same temporal hypothesis and notation conventions as in the previous example.

$\text{MEETS}(I_0, I_1) \wedge \text{MEETS}(I_1, I_2)$
$\text{Holds}_{on}(coupled(n1, car1), I_0)$
$\text{Do}_{on}(setthrottle(n1), I_1)$
$\text{Do}_{on}(activating(n1), I_2)$
$(SE1)\ \forall_{\mathscr{W}} n \forall_{\mathscr{I}} I, I' (\text{Do}_{on}(setthrottle(n), I) \succ\!\!- \text{Occurs}_{on}(move(n), I')) \wedge \text{MEETS}(I, I')$
$(SE2)\ \forall_{\mathscr{W}} n \forall_{\mathscr{I}} I (\text{Do}_{on}(activating(n), I) \succ\!\!- \text{Occurs}_{on}(activate(n), I))$

$(SE3)\ \forall_{\mathscr{W}} n, c \forall_{\mathscr{I}} I', I'', I (\text{Holds}_{on}(coupled(n, c), I) \wedge \text{Occurs}_{on}(move(n), I') \wedge$
$\text{Occurs}_{on}(activate(n), I'') \wedge \neg(I' \lhd\!\rhd I'') \wedge \text{MEETS}(I, conj(I', I'')$

$$\succ\!\!- \text{Occurs}_{on}(uncouple(n, c), conj(I', I'')))$$

where $conj(I', I'')$ is the common subinterval between intervals I' and I''.

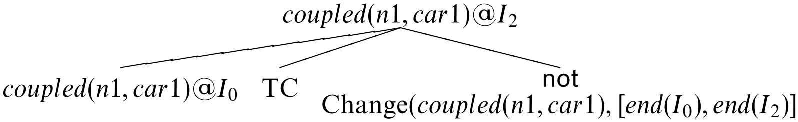


Fig. 10. Argument tree for $coupled(n1, car1)@I_2$

$(SE4)$
$\forall n, c \forall_{\mathscr{I}} I(\mathrm{Occurs}_{on}(uncouple(n, c), I) \succ\!\!- \neg\mathrm{Holds}_{on}(coupled(n, c), I') \wedge \mathrm{MEETS}(I, I')$

$(SE5)\ \forall_{\mathscr{A}} a \forall_{\mathscr{I}} I(\mathrm{Do}_{on}(a, I) \leftrightarrow$

$$((a = \mathrm{Do}_{on}(setthrottle(n1), I) \wedge I = I_1) \vee (a = \mathrm{Do}_{on}(activating(n1), I)) \wedge I = I_2)))$$

We have an argument to support $\mathrm{Holds}_{on}(coupled(n1, car1), I_3)$ by persistency using Theorem 1 as follows:

$$\begin{aligned} A = \langle \{ & \mathrm{Holds}_{on}(coupled(n1, car1), I_0) \wedge (end(I_0) \leqslant begin(I_3)) \wedge \\ & \mathsf{notChange}^{+-}_{in}(coupled(n1, car1), [end(I_0), end(I_3)]) \succ\!\!- \\ & \qquad \mathrm{Holds}_{on}(coupled(n1, car1), I_3)\}, \\ & \qquad \mathrm{Holds}_{on}(coupled(n1, car1), I_3) \rangle \end{aligned}$$

This argument is represented in the tree of Fig. 10.

If setting the throttle occurs later, for instance $\mathrm{Do}(setthrottle(n1), I_2)$, it is no longer possible to use $(SE4)$ and the coupling persists. But, if the moving event occurs during I_2 the decoupling can be done:

$$\begin{aligned} B_1 = \langle \{ & \mathrm{Do}_{on}(setthrottle(n), I_1) \succ\!\!- \mathrm{Occurs}_{on}(move(n), I_2)\}, \\ & \mathrm{Occurs}_{on}(move(n), I_2) \rangle \end{aligned}$$

From $\mathrm{Do}(activating(n1), I_2)$ and $(SE2)$ we can obtain $\mathrm{Occurs}(activate(n1), I_2)$:

$$\begin{aligned} B_2 = \langle \{ & \mathrm{Do}_{on}(activating(n), I_2) \succ\!\!- \mathrm{Occurs}_{on}(activate(n), I_2)\}, \\ & \mathrm{Occurs}_{on}(activate(n), I_2) \rangle \end{aligned}$$

By persistency it could be assumed $\mathrm{Holds}(coupled(n1, car1), I_2)$:

$$\begin{aligned} B_3 = \langle \{ & \mathrm{Holds}_{on}(coupled(n1, car1), I_0) \wedge \mathrm{MEETS}(I_0, I_1) \wedge \\ & \mathsf{notChange}^{+-}_{in}(coupled(n1, car1), I_1) \succ\!\!- \mathrm{Holds}_{on}(coupled(n1, car1), I_1)\}, \\ & \mathrm{Holds}_{on}(coupled(n1, car1), I_1) \rangle \end{aligned}$$

By using arguments B_1, B_2, B_3 and $SE3$ we could support $\mathrm{Occurs}(uncouple(n1, car1), I_2)$:

$$\begin{aligned} B_4 = \quad & \langle \{ \mathrm{Holds}_{on}(coupled(n1, car1), I_1) \wedge \\ & \mathrm{Occurs}_{on}(move(n1), I_2) \wedge \\ & \mathrm{Occurs}_{on}(activate(n1), I_2) \wedge \\ & \neg(I_2 \lhd\!\rhd I_2) \wedge \mathrm{MEETS}(I_1, I_2) \succ\!\!- \quad \mathrm{Occurs}_{on}(uncouple(n1, car1), I_2)\}, \\ & \mathrm{Occurs}_{on}(uncouple(n1, car1), I_2) \rangle \end{aligned}$$

Finally, by $SE4$ we can build the following argument:

$$\begin{aligned} B = \quad & \langle \{ \mathrm{Occurs}_{on}(uncouple(n1, car1), I_2) \succ\!\!- \quad \neg\mathrm{Holds}_{on}(coupled(n1, car1), I_3)\}, \\ & \neg\mathrm{Holds}_{on}(coupled(n1, car1), I_3) \rangle \end{aligned}$$

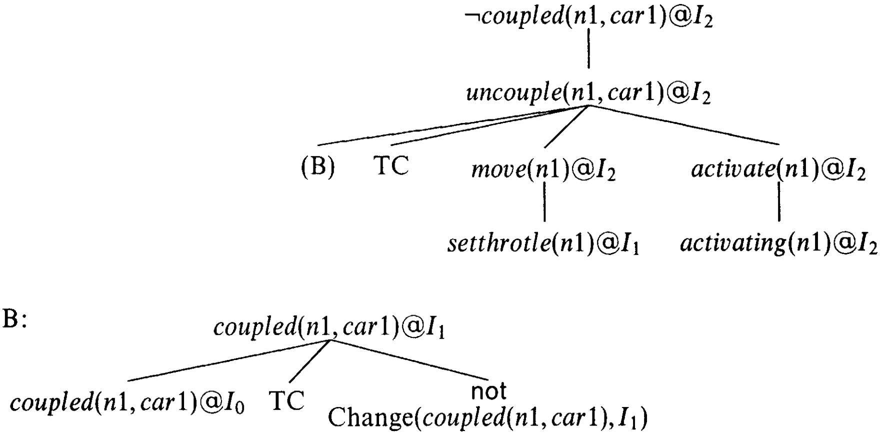


Fig. 11. Argument tree for $\neg coupled(n1, car1)@I_2$.

This argument is shown in the tree of Fig. 11.

It can be concluded that $B \bowtie_{\mathbb{T}} A$, $B \succ_{tspec} A$ and $B \gg_{tdef} A$. Then B allows to justify $\neg\mathrm{Holds}(coupled(n1, car1), I_3)$.

6. Conclusions

An extension of the argumentation system offered in Augusto and Simari (1994, 1999) was presented. The new proposal formalizes defeasible temporal reasoning allowing both instants and intervals as temporal references. Previous work on this line could be used just with one of this kind of reference. Using both instants and intervals, we avoid problems addressed elsewhere (Galton, 1990) to other purely interval-based proposals (Allen, 1984; Ferguson and Allen, 1994; Ferguson, 1995).

The many-sorted logic has clearly specified syntax, semantics and inference rules. Further details of these temporal and monotonic layers of the proposal can be seen in (Augusto, 1998) or (Augusto, 2000), where some temporal concepts are developed more deeply. In particular, Augusto (2000) includes a comparison between the temporal logic we used as a basement in this proposal and other works in the literature which consider instants and intervals.

We provided explicit time-based temporal reasoning but also event-driven reasoning in an integrated framework. Also, as argued in Augusto (2000), the combination of instants and intervals is twofold: theoretical and practical. From a theoretical perspective it allows a solution to problems of continuous change (Galton, 1990) and the dividing instant problem (Vila, 1994). On the other side, the solution to some problems can be based on a combination of instants and intervals avoiding complexity issues through polynomial-time algorithms (Ladkin and Maddux, 1988; Meiri, 1992) instead of exponential-time algorithms forced by using a purely interval-based proposal (Allen, 1981; Vilain and Kautz, 1986).

Other important issues on the temporal reasoning perspective like persistency and causality were considered. We do not consider these as our last word on the subject but it is useful to show how the basic temporal theory could be extended in some key directions. The system's behavior was illustrated by means of well-known problems of the literature.

This system has not been implemented yet but there are good starting points, like an implementation for a non-temporal argumentation system (García, 1997) and typed logic-programming style languages that can be useful to build a prototype. Algorithms for a proof procedure of a many-sorted first-order logic with equality like that used here in $\mathscr{L}^{\mathbb{T}}$ can be obtained in Gallier (1987). This is also the recommended source for the reader interested in the meta-theoretical properties of many-sorted logics like those used in this work.

The argumentation system itself has been extensively studied and several developments are being conducted taking that proposal as a basis (Chesñevar et al., 1998). A substantial study has been made considering the wide range of applications that this kind of system has (Carbogim et al., 2000), but most efforts have been addressed to non-temporal systems. As a result, their capabilities for solving problems in dynamic domains is very limited in the best case. Two proposals have been under development during recent years to mitigate this absence with different goals in mind. In Ferguson (1995) an argumentation framework is considered to formalize dialogue conventions. Conversely, in Augusto (1998) research has been conducted on a more theoretical basis, getting a more in-detail formulation for both the temporal and the argumentation layers of the proposal. We think this formulation improves the understanding of this kind of system both from the theoretical and programmer's perspective.

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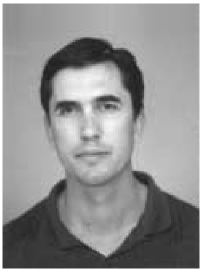

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