

Summary

This dissertation consists of two parts.

Part I concentrates on the axiomatizability of process algebras, mostly on two basic process algebras BCCSP and BCCS.

Chapter 3 presents two meta-theorems regarding the axiomatizability. The first one concerns the relationship between preorders and equivalences. We show that the same algorithm proposed by Aceto *et al.* and de Frutos Escrig *et al.* for concrete semantics, which transforms an axiomatization for a preorder to the one for the corresponding equivalence, applies equally well to weak semantics. This makes it applicable to all 87 preorders surveyed in the “linear time – branching time spectrum II” that are at least as coarse as the ready simulation preorder. We also extend the scope of the algorithm to infinite processes, by adding recursion constants. The second meta-theorem concerns the relationship between concrete and weak semantics. For any semantics which is not finer than failures or impossible futures semantics, we provide an algorithm to transform an axiomatization for the concrete version to the one for the weak counterpart. As an application of this algorithm, we derive ground- and ω -complete axiomatizations for weak failure, weak completed trace, weak trace preorders.

Chapter 4 settles the remaining open questions regarding the existence of ω -complete axiomatizations in the setting of the process algebra BCCSP for all the semantics in the linear time – branching time spectrum I, either positively by giving a finite, sound and ground-complete axiomatization which turns out to be ω -complete, or negatively by proving that such a finite basis of the equational theory does not exist. We prove that in case of a finite alphabet with at least two actions, failure semantics affords a finite basis, while for ready simulation, completed simulation, simulation, possible worlds, ready trace, failure trace and ready semantics, such a finite basis does not exist. Completed simulation semantics also lacks a finite basis in case of an infinite alphabet of actions.

Chapter 5 investigates the (in)equational theories of concrete and weak impossible futures semantics over the process algebras BCCSP and BCCS. We present a finite, sound, ground-complete axiomatization for BCCSP modulo the concrete impossible futures preorder, which implies a finite, sound, ground-complete axiomatization for BCCS modulo the weak impossible futures preorder. By contrast, we prove that no finite, sound axiomatization for BCCS modulo the weak impossible futures equivalence is ground-complete, and this negative result carries over to the concrete case. If the alphabet of actions is infinite, then the aforementioned ground-complete axiomatizations are shown

to be ω -complete. However, if the alphabet is finite and nonempty, we prove that the inequational (resp. equational) theories of BCCSP and BCCS modulo the impossible futures preorder (resp. equivalence) lack such a finite basis. Finally, we show that the negative result regarding impossible futures equivalence extends to all n -nested impossible futures equivalences for $n \geq 2$, and to all n -nested impossible futures preorders for $n \geq 3$.

Chapter 6 studies the equational theory of bisimulation equivalence over the process algebra BCCSP_Θ , i.e., BCCSP extended with the priority operator Θ of Baeten *et al.* It is proven that, in the presence of an infinite set of actions, bisimulation equivalence has no finite, sound, ground-complete axiomatization over that language. This negative result applies even if the syntax is extended with an arbitrary collection of auxiliary operators, and motivates the study of axiomatizations using equations with action predicates as conditions. In the presence of an infinite set of actions, it is shown that, in general, bisimulation equivalence has no finite, sound, ground-complete axiomatization consisting of equations with action predicates as conditions over BCCSP_Θ . Finally, sufficient conditions on the priority structure over actions are identified that lead to a finite, sound, ground-complete axiomatization of bisimulation equivalence using equations with action predicates as conditions.

Part II concentrates on the verification of probabilistic real-time systems.

Chapter 7 studies the following problem: given a continuous-time Markov chain \mathcal{C} , and a linear real-time property provided as a deterministic timed automaton \mathcal{A} , what is the probability of the set of paths of \mathcal{C} that are accepted by \mathcal{A} ? It is shown that this set of paths is measurable and computing its probability can be reduced to computing the reachability probability in a piecewise deterministic Markov process. The reachability probability is characterized as the least solution of a system of integral equations and is shown to be approximated by solving a system of partial differential equations. For the special case of single-clock deterministic timed automata, the system of integral equations can be transformed into a system of linear equations where the coefficients are solutions of ordinary differential equations.

Chapter 8 focuses on probabilistic timed automata, an extension of timed automata with discrete probabilistic branchings. As the region construction of these automata often leads to an exponential blow-up over the size of original automata, reduction techniques are of the utmost importance. In this chapter, we investigate probabilistic time-abstracting bisimulation (PTAB), an equivalence notion that abstracts from exact time delays. PTAB is proven to preserve probabilistic computation tree logic. The region equivalence is a (very refined) PTAB. Furthermore, we provide a non-trivial adaptation of the traditional partition-refinement algorithm to compute the quotient under the PTAB. This algorithm is symbolic in the sense that equivalence classes are represented as polyhedra.