Problems 1:
Options and rational pricing

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Question 1

Google Inc. traded at $S = \$542$ on 23 Jan 2015. What should be the forward price of each share in three months time, if the bank lends money at the riskless rate of $r = 3\%$ APR? What if the stock pays dividends at $s = 5\%$ APR? Work out your answers assuming that the dividends are continuously compounded. What if both the stock and the bank compound the dividends once in three months?

Answer: *If you borrow $S = \$542$ from the bank to buy one Google share, then after $T - t$ years time you have to return to the bank $e^{r(T-t)}S$. Thus, the forward price should not cost more (otherwise you make a riskless profit). It also should not cost less, because otherwise you can borrow and sell the stock now, invest S amount in the bank, and then buy the stock for less than $e^{r(T-t)}S$ at T and return the stock (generating a riskless profit). For continuously compounded dividends the fair price is computed as:*

$$F(t, T) = e^{r(T-t)}S(t)$$

With $r = .03$, $T - t = 3/12 - 0 = .25$ and $S(0) = \$542$ the price is:

$$F(0, 0.25) = e^{.03\times.25} \times \$542 = \$546$$

If the stock pays dividends, then the effective rate of return is the difference $r - s$. Indeed, to deliver one share in the future, one needs to borrow from the bank and buy $e^{-s(T-t)}S$ of stock now. The repayment (and hence the forward price) will be

$$F(t, T) = e^{r(T-t)}e^{-s(T-t)}S(t) = e^{(r-s)(T-t)}S(t) = e^{-.02\times.25}\times\$542 = \$539.30$$

If the dividends are compounded n times per year, then we should use $(1 + r/n)^{n(T-t)}$ instead of $e^r(T - t)$ in the formulae. Thus, with $T - t =$

$3/12 = .25$ *and* $n = 4$ *(once in three months means four times per year), we have:*

$$F(t,T) = \left(1 + \frac{(r-s)}{n}\right)^{n(T-t)} S(t) = \left(1 - \frac{.02}{4}\right)^{4/4} \$542 = \$539.29$$

Question 2

BAE was listed at $S(t) = £519$ on London Stock Exchange on closing of 23 Jan 2015. Work out the riskless interest rate r under the conditions of no arbitrage and no dividends paid, if BAE forward price in 6 months is $F(t,T) = £540$.

Answer: *Inverting the formula* $F(t,T) = e^{r(T-t)}S(t)$ *gives:*

$$r = \frac{1}{T-t} \ln\left(\frac{F(t,T)}{S(t)}\right)$$

with $T - t = 6/12 = 1/2$, *we have*

$$r = 2 \times \ln(£540/£519) \approx 0.079$$

or 7.9% APR.

Question 3

The final payoff for a long call and a long put options with strike price K are respectively:

$$V_{\text{call}}(T) = \max[0, S(T) - K]\,, \quad V_{\text{put}}(T) = \max[0, K - S(T)]$$

Modify these formulae to take into account the transaction costs (T-costs). That is, taking into account the prices of options at $t \leq T$. What are the corresponding functions for short call and put?

Answer: *Simply subtract the costs of options from the final payoffs. Thus, denoting by* $C(t)$ *and* $P(t)$ *the costs of long call and put options, we get:*

$$V_{\text{call}}(T) = \max[0, S(T) - K] - C(t)\,, \quad V_{\text{put}}(T) = \max[0, K - S(T)] - P(t)$$

When shorting options, the writer (the seller) receives $C(t)$ *for a call and* $P(t)$ *for a put. The final payoffs are simply negative payoff for long options:*

$$V_{\text{call}}(T) = C(t) - \max[0, S(T) - K]\,, \quad V_{\text{put}}(T) = P(t) - \max[0, K - S(T)]$$

Question 4

Trading options depends on the trader's forecast (or view) about the future price of the underlying stock $S(T)$. This view maybe optimistic (bullish), if the trader believes the stock will rise, or pessimistic (bearish), if the trader believes the stock will fall. Describe which views correspond to long call, short call, long put, short put option? At which stock price $S(T)$ does the trader break even in each case?

Answer:

Long call : *The trader buys an option to buy stock for K. The view is optimistic, because the trader believes that $S(T)$ will rise above K. The buyer breaks even if $S(T) = K + C$, where C is the cost of the call option.*

Short call : *The trader (writer) sells an option to buy stock for K. The view is pessimistic, because the writer believes that $S(T)$ will fall below K. The writer breaks even if $S(T) = K + C$, where C is the cost of the call option.*

Long put : *The trader buys an option to sell stock for K. The view is pessimistic, because the trader believes that $S(T)$ will fall below K. The buyer breaks even if $S(T) = K - P$, where P is the cost of the put option.*

Short put : *The trader (writer) sells an option to sell stock for K. The view is optimistic, because the writer believes that $S(T)$ will rise above K. The writer breaks even if $S(T) = K - P$, where C is the cost of the put option.*

This is summarised in the table below.

	Call	*Put*
Long	*Optimistic,* $S(T) \geq K + C$	*Pessimistic,* $S(T) \leq K - P$
Short	*Pessimistic,* $S(T) \leq K + C$	*Optimistic,* $S(T) \geq K - P$

Question 5

ROSNEFT oil company closed at $S(t) = \$3.88$ per share on 23 Jan 2015, which is almost twice lower than the 52 Week High of \$7.4. Compute and compare the profits and returns on investment in 1000 shares with an equivalent amount on call options with the strike price $K = \$5$ expiring in $T - t = 6$ months and priced $C = \$.50$ per call, if the share price rises back to $S(T) = \$7.4$. At which stock price $S(T)$ does the investment in shares give the same return as in the call options?

Answer: *The profit from investment in 1000 shares is:*

$$1000 \times (S(T) - S(t)) = 1000 \times (\$7.4 - \$3.88) = \$3,520$$

and the rate of return is:

$$\frac{S(T) - S(t)}{S(t)} = \frac{\$3.52}{\$3.88} \approx .91$$

The budget of $1000 \times \$3.88 = \$3,880$ *allows one to buy* $\$3,880/\$0.50 = 7,760$ *call options. Then exercising the option at* $K = \$5$ *and selling stock for* $S(T) = \$7.4$ *would generate the profit:*

$$7,760 \times (\$7.4 - \$5 - \$.5) = \$14,744$$

producing the rate of return return of

$$\frac{S(T) - K - C}{C} = \frac{\$7.4 - \$5 - \$.5}{\$.5} \approx 3.80$$

or 380%.

Using the equality of returns:

$$\frac{S(T) - S(t)}{S(t)} = \frac{S(T) - K - C}{C}$$

gives the following formula for the required stock price:

$$S(T) = \frac{K}{1 - \frac{C}{S(t)}}$$

Substituting $K = \$5$, $S(t) = \$3.88$, $C = \$.5$ *gives the return approx.* 48%.

Question 6

Given $S(t) = \$3.88$ and $C(t) = \$.5$ for a call option expiring in 6 months with strike price $K = \$5$, compute the price of a put option, if the riskless rate is 1% APR.

Answer: *Using the call-put parity*

$$C(t) - P(t) = S(t) - e^{-r(T-t)}K$$

the price of put is given by the formula:

$$P(t) = C(t) - S(t) + e^{-r(T-t)}K = \$.5 - \$3.88 + e^{-.01/2} \times \$5 \approx \$1.6$$

Question 7

What should be the stock price $S(t)$ to make the cost of call option $C(t)$ higher than the cost $P(t)$ of a put option, if the strike price is $K = \$5$ expiring in 6 months for both options, and the riskless rate is 1% APR?

Answer: *Using the call-put parity gives*

$$C(t) - P(t) \geq 0 \quad \Longleftrightarrow \quad S(t) - e^{-r(T-t)}K \geq 0$$

which gives the condition:

$$S(t) \geq e^{-r(T-t)}K = e^{-.01/2} \times \$5 \approx \$4.96$$

Question 8

What is the present value C of a call and P of a put option with strike price $K = \$5$ expiring in 3 months, if the riskless rate is 2% APR, and you know with certainty that the stock price will be $S(T) = \$4$ or lower? What can be said about the current stock price $S(t)$? Does this conclusion about $S(t)$ still hold, if the condition $S(T) \leq \$4$ is based on an insider information?

Answer: *We use the formula for the present value of the expected payoff:*

$$V(t) = e^{-r(T-t)}\mathbb{E}\{V(T)\}$$

where $V(T)$ is the final payoff (i.e. $V(T) = \max[0, S(T) - K]$ for a long call and $V(T) = \max[0, K - S(T)]$ for a long put). If we know that $S(T) \leq \$4$ with certainty, then for $K = \$5$ the expected payoff for call is $\mathbb{E}\{V(T)\} \leq \max[0, \$4 - \$5] = \$0$, and for put $\mathbb{E}\{V(T)\} \geq \max[0, \$5 - \$4] = \$1$. Taking into account the interest rate $r = .02$ and $T - t = 1/4$, the present values are:

$$C(t) \leq e^{-.02/4} \times \$0 = \$0\,, \qquad P(t) \geq e^{-.02/4} \times \$1 \approx \$.995$$

Stock price can be derived using the call-put parity:

$$S(t) = e^{-r(T-t)}K + C(t) - P(t)$$

Using $C(t) \leq \$0$ and $P(t) \geq \$.995$ we conclude that

$$S(t) \leq e^{-.02/4} \times \$5 + \$0 - \$.995 \approx \$3.98$$

This conclusion is no longer valid, because the call-put parity formula is based on the assumption of no arbitrage, which is violated if one has an insider information (e.g. if you know about some extremely negative news about the stock to be published before the expiration date).