Problems 3:
Stochastic processes and nowhere differentiable functions

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Question 1

Which of the functions below are infinitely many times differentiable (i.e. analytic)?

$$f(x) = x\,, \quad f(x) = \frac{x^2}{2}\,, \quad f(x) = e^x\,, \quad f(x) = \ln x\,, \quad f(x) = \cos x$$

Question 2

For each function above, write its Taylor series expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

Expand $f(x) = \ln x$ at $x_0 = 1$, and all other functions at $x_0 = 0$ (this is called the Maclaurin series).

Question 3

Why does the existence of a derivative $f'(x)$ at $x = x_0$ requires $f(x)$ to be continuous at $x = x_0$? Hint: use the 'limits' definition of continuity ($\lim_{x\to x_0} f(x) = f(x_0)$).