

Problems 1: Options and rational pricing

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Question 1

Google Inc. traded at $S = \$542$ on 23 Jan 2015. What should be the forward price of each share in three months time, if the bank lends money at the riskless rate of $r = 3\%$ APR? What if the stock pays dividends at $s = 5\%$ APR? Work out your answers assuming that the dividends are continuously compounded. What if both the stock and the bank compound the dividends once in three months?

Question 2

BAE was listed at $S(t) = \pounds 519$ on London Stock Exchange on closing of 23 Jan 2015. Work out the riskless interest rate r under the conditions of no arbitrage and no dividends paid, if BAE forward price in 6 months is $F(t, T) = \pounds 540$.

Question 3

The final payoff for a long call and a long put options with strike price K are respectively:

$$V_{\text{call}}(T) = \max[0, S(T) - K], \quad V_{\text{put}}(T) = \max[0, K - S(T)]$$

Modify these formulae to take into account the transaction costs (T-costs). That is, taking into account the prices of options at $t \leq T$. What are the corresponding functions for short call and put?

Question 4

Trading options depends on the trader's forecast (or view) about the future price of the underlying stock $S(T)$. This view maybe optimistic (bullish), if the trader believes the stock will rise, or pessimistic (bearish), if the trader believes the stock will fall. Describe which views correspond to long call, short call, long put, short put option? At which stock price $S(T)$ does the trader break even in each case?

Question 5

ROSNEFT oil company closed at $S(t) = \$3.88$ per share on 23 Jan 2015, which is almost twice lower than the 52 Week High of $\$7.4$. Compute and compare the profits and returns on investment in 1000 shares with an equivalent amount on call options with the strike price $K = \$5$ expiring in $T - t = 6$ months and priced $C = \$0.50$ per call, if the share price rises back to $S(T) = \$7.4$. At which stock price $S(T)$ does the investment in shares give the same return as in the call options?

Question 6

Given $S(t) = \$3.88$ and $C(t) = \$0.5$ for a call option expiring in 6 months with strike price $K = \$5$, compute the price of a put option, if the riskless rate is 1% APR.

Question 7

What should be the stock price $S(t)$ to make the cost of call option $C(t)$ higher than the cost $P(t)$ of a put option, if the strike price is $K = \$5$ expiring in 6 months for both options, and the riskless rate is 1% APR?

Question 8

What is the present value C of a call and P of a put option with strike price $K = \$5$ expiring in 3 months, if the riskless rate is 2% APR, and you know with certainty that the stock price will be $S(T) = \$4$ or lower? What can be said about the current stock price $S(t)$? Does this conclusion about $S(t)$ still hold, if the condition $S(T) \leq \$4$ is based on an insider information?