

MIDDLESEX UNIVERSITY

EXAMINATION PAPER

Mock exam paper (May)

MSO4112

Pricing and Stochastic Calculus

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Time allowed:	2 Hours
Total number of questions:	Three
Instructions to candidates:	Answer any TWO questions. Each question is worth a maximum of 25 marks. The total (out of 50) will be converted to a percentage after marking.
Materials provided:	None
Equipment permitted:	Non-programmable calculator
Total number of pages:	3 including this page

EXAM PAPER CANNOT BE REMOVED FROM THE EXAM ROOM

No books, paper or electronic devices are permitted to be brought into the examination room other than those specified above.

Candidates are warned that credit cannot be given for work that is illegible.

Question 1

- a) Describe the main types of options. Explain the terms ‘in the money’, ‘out of money’ and ‘at the money’ for each type of European options. [10 marks]
- b) ROSNEFT oil company closed at $S(t) = \$3.88$ per share on 23 Jan 2015, which is almost twice lower than the 52 Week High of $\$7.4$. Compute and compare the profits and returns on investment in 1000 shares with an equivalent amount on call options with the strike price $K = \$5$ expiring in $T - t = 6$ months and priced $C = \$0.50$ per call, if the share price rises back to $S(T) = \$7.4$. At which stock price $S(T)$ does the investment in shares give the same return as in the call options? [10 marks]
- c) Google Inc. traded at $S = \$542$ on 23 Jan 2015. What should be the forward price of each share in three months time, if the bank lends money at the riskless rate of $r = 3\%$ APR? What if the stock pays dividends at $s = 5\%$ APR? Work out your answers assuming that the dividends are continuously compounded. What if both the stock and the bank compound the dividends once in three months? [5 marks]

Question 2

- a) Give a definition of Markov stochastic process. What is a continuous Markov process? [7 marks]
- b) What is the power spectrum of a stationary process with auto-correlation $k(\tau) = \sigma^2 e^{-\beta|\tau|}$ with $\sigma^2 = 1$ (i.e. Gaussian exponentially correlated process)? For which values of β can we model such a process by a standard white noise? [7 marks]
- c) Let $y = f(x)$ be a twice-differentiable function, and $x(t)$ be a stochastic process described by a stochastic differential equation $dx(t) = x(t) dt + 7 dw(t)$. Derive Ito rule for the differential $dy(t)$ and write the differential of $y(t) = e^{x(t)}$. [7 marks]
- d) Describe the reason why standard rules of differential calculus cannot be used? [4 marks]

Question 3

- a) Let $S = £700$ be the stock price, which after time T can only change up $S_u = £800$ or down $S_d = £500$. Compute the risk-neutral probabilities of up move S_u and down move, assuming that the risk-free interest rate $r = 0$. How will these probabilities change, if $r > 0$? [5 marks]
- b) Assuming $N = 2$ step binomial model, price European call option on stock described above with the strike price $K = £750$ and expiring in $T = 1$ year. Describe how the value changes as a function of the strike price. What is the price of the corresponding put? What if the number N in the binomial model grows to infinity? [10 marks]

- c) The following equations are the Black-Scholes prices of call and put options with stock S paying dividends at constant continuous rate ρ :

$$\begin{aligned}C(S, t) &= e^{-\rho(T-t)}SN(d_1) - e^{-r(T-t)}KN(d_2) \\P(S, t) &= e^{-r(T-t)}KN(-d_2) - e^{-\rho(T-t)}SN(-d_1)\end{aligned}$$

where

$$d_1 = \frac{\ln(S/K) + (r - \rho + \sigma^2/2)(T - t)}{\sigma\sqrt{T - t}}, \quad d_2 = d_1 - \sigma\sqrt{T - t}$$

Differentiate over S to derive the equations for Δ and Γ . Hint: use the fact that the CDF of the standard normal distribution ($\mu = 0, \sigma^2 = 1$) is $N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-s^2/2} ds$, and its derivative is $dN(x)/dx = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$

[10 marks]