Middlesex University London

Faculty of Science and Technology

MSO4112

Pricing and Stochastic Calculus

Module Leader: R.V.Belavkin

Term: AY (2019)

Duration of the module: 12 weeks

Document version: 1

Other formats available

This handbook is available in a large print format. If you would like a large print copy or have other requirements for the handbook, please contact the Disability Support Service (disability@mdx.ac.uk, +44 (0)20 8411 4945).

Disclaimer

The material in this handbook is as accurate as possible at the date of production. You will be notified of any minor changes. If there are any major changes to the module you will be consulted prior to the changes being confirmed. Please check the version number on the front page of this handbook to ensure that you are using the most accurate information.

Other documents

Your module handbook should be read and used alongside your programme handbook and the information available to all students on My Learning, including the Academic Regulations. Your programme handbook can be found on the My Learning programme page.

Contents

1 Module Introduction

This module aims to teach students to use pricing theory for derivatives, such as options, futures and forwards, using a risk-neutral probability and stochastic differential equations (SDEs). It explores discrete and continuous time models of stochastic processes with applications to pricing, such as the Black-Scholes equation for options pricing. In addition to theoretical exploration, the module introduces students to the main numerical methods, such as Monte-Carlo simulations, for modelling and solving option pricing problems.

2 The Module Team

Roman V. Belavkin	
	
Role:	Module Leader
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3 Staff Student Communication

Students may contact staff via e-mail, phone, by dropping in to staff office hours, and by making an appointment to see them outside office hours. Staff will contact students by e-mail, phone, the My Learning module page and via lectures and seminars.

The team may send urgent group and/or individual messages about the module to you by email, so it is important that you read your University email regularly. All staff have office hours, it is not necessary to book an appointment during these hours, you just need to drop-in.

In the first instance problems should be dealt with by talking to a member of the module team. You can give feedback on this module to the module leader, your Student Voice Leader, to your personal tutor, and through the end of module evaluation survey.

4 Module Overview

4.1 Aims

The module aims to give students both basic understanding of the Black-Scholes theory together with the theory of stochastic processes on which it is based and some practical skills

for using the Black-Scholes formula to price options. More importantly, students will analyse the main theoretical assumptions of the theory in order to understand its limitations. Practical work will equip students with methods that will enable them to test and verify whether these assumptions hold in real market data.

4.2 Learning outcomes

Knowledge On completion of this module the successful student will be able to:

1. Formulate the main principles and assumptions of rational pricing
2. Define the main types of stochastic differential equations (SDE) and integrals
3. Reason about the main principles of application of SDE to pricing of financial assets.
4. Differentiate between several techniques for solving SDE

Skills This module will call for the successful student to:

1. Model a specific pricing problem by an appropriate SDE
2. Analyse data to estimate parameters of stochastic processes used in an SDE
3. Obtain analytical solutions to basic SDEs
4. Use computer simulation to obtain numerical solutions to an SDE

4.3 Syllabus

- Options and rational pricing
- Binomial pricing
- Stochastic processes and nowhere differentiable functions
- Gaussian white noise and Wienner process
- Continuous Markov process and the diffusion equation
- Stochastic differential equations and integrals
- Ito rule
- The Black-Scholes theory

4.4 Learning and teaching strategy

4.5 Assessment scheme

Formative assessment will consist of guided and independent activities throughout the module. Feedback will be provided in subsequent labs and seminars or on-line.

Summative assessment consists of two components selected in order to ensure students demonstrate an overall understanding of relevant concepts and techniques as well as the ability to apply and critique them in appropriate contexts.

The summative assessment components are:

1. Individual report (50%) students will be given data to analyse and produce a numerical solution to a pricing problem. The report (between 4–6 A4 pages) should discuss the model, calculations, results and discussion of the findings. This will address learning outcomes 5, 6, 8 (Week 11).
2. Two hour unseen examination (50%) assessing key concepts throughout the module. The paper will consist of a choice of questions, which will address learning outcomes 1, 2, 3, 4, 7 (Examination Period)

In order to pass this module students must achieve a grade 18 or better in each summative assessment component.

4.6 Assessment Weighting

1. Unseen two-hour examination, 50% of the total mark.
2. Coursework, 50% of the total mark as three assessed assignments.

4.7 Learning hours

The learning hours for each credit point is 10 hours, and for a 15-credit module this equates to 150 hours. Therefore, if a module has time-tabled activities i.e. lecture/seminar/lab, of 3 hours per week for a 12 week period (total of 36 hours), then the out-of-class study commitment expected of students is 9 hours per week or 114 hours in total.

Research Ethics

- The teaching, learning, assessment and research activities undertaken in this module have been considered and are not likely to require ethical approval.
- However, please seek advice if undertaking the module entails carrying out any research activities involving human participants, human data, animals/animal products, precious artefacts, materials or data systems. If you submit work that includes data

gathered from or about people, this may be treated as academic misconduct and could lead to fail grade being awarded.

- Research ethics approval seeks to ensure all research is designed and undertaken according to certain principles of ethical research. These include:

  1. Primary concern must be given to the safety, welfare and dignity of participants, researchers, colleagues, the environment and the wider community
  2. Consideration of risks should be undertaken before research commences with the aim of minimising risks to those involved – i.e. human participants or animal subjects, colleagues, the environment and the wider community, as well as actual or potential risks to those directly or indirectly affected by the research.
  3. Informed consent should be freely given by participants, and by a trained person when collecting or analysing human tissue (details on accessing and completing online training for gaining informed consent for HTA purposes can be found below in Section 8).
  4. Respect for the privacy, confidentiality and anonymity of participants
  5. Consideration of the rights of people who may be vulnerable (by virtue of perceived or actual differences in their social status, ethnic origin, gender, mental capacities, or other such characteristics) who may be less competent or able to refuse to give consent to participate
  6. Researchers have a responsibility to the general public and to their profession; as such they should balance the anticipated benefits of their research against potential harm, misuse or abuse which must be avoided
  7. Researchers must demonstrate the highest standards of ethical conduct and research integrity. They must work within the limits of their skills, training and experience, and refrain from exploitation, dishonesty, plagiarism, infringement of intellectual property rights and the fabrication of research results. They should declare any actual or potential conflicts of interest, and where necessary take steps to resolve them.
  8. When using human tissues for research, Human Tissue Act and Human Tissue Authority (HTA) requirements must be met. Please contact the relevant designated person (DP) in your department or the HTA Designated Individual (DI) (Dr Lucy Ghali - L.Ghali@mdx.ac.uk). Further information is provided below in the section: “Human Tissue Authority Information”, see “Governance Structure” document and SOPs etc.
  9. Research should not involve any illegal activity, and researchers must comply with all relevant laws.

- For more information about ethics go to the Middlesex Online Research Ethics (MORE) system which has information and guidance to help you meet the highest standards of ethical research using this link: `https://MOREform.mdx.ac.uk`

- Information and further guidance on how to complete a research ethics application form (e.g., video guides and templates) can be found on the MORE MyLearning site*: `http://mdx.mrooms.net/enrol/index.php?id=12277` (Log in required) *Middlesex University Definition of Research document can be located on this site.

5 Learning Resources

Each topic of the syllabus supported by lecture slides, the handouts for which are available on *MyLearning* as well as the module leader's webpage:

`http://www.eis.mdx.ac.uk/staffpages/rvb/teaching/MSO4112/`

There are also sets of problems that students have to solve each week. Solutions and model answers to these problems are published one later. You should always try to solve the problems before the solutions are published or before looking up the solutions. Do not afraid making mistakes. It is normal to make mistakes when learning, and you will learn more from errors.

The key reading materials for the module are listed in the Reading list:

`http://readinglists.mdx.ac.uk`

Please, refer to the Lecture plan for specific chapters in each week.

6 Making the most of this module

One of the key elements to successfully completing this module is not only attending the lectures and labs, but also regularly (each week) working on the sets of problems that are given as homework. These problems are designed to deepen the understanding of each topic in the module and prepare you for the next topic. Solving mathematical problems is a skill that requires regular training spread over time, similar to training muscles in a gym. In addition, it is important to complete practical tasks that are discussed in the labs. These will help you prepare better for your coursework.

Participation and engagement

This module is designed as a combination of contact sessions and independent study. This means you should attend all the allocated sessions, and you should work on your own outside them. Students are expected to take an active part in all learning sessions (lectures and lab sessions).

Student attendance is monitored during *lectures* and *labs*, and any unexplained absences will be followed up via e-mail. If for any reason you are unable to attend a session, you please inform the module leader.

To make the most of this module please complete the following every week

- Read through the lecture notes making a note of any points you need to discuss with your tutor.
- Complete the set of problems for each week before the next session, making a note of any points you need to discuss with your tutor.
- Complete practical work and tasks on a computer shown in the Lab.

- Complete further reading from the core text online.

The module team is committed to support you and your fellow students whilst you undertake this module. In order for you to get the most out of sessions, you need to come prepared and ready to contribute. Please ensure that any work set by the team has been completed before workshops. After each class please review what has been covered and make a note of anything you would like clarification on.

It is important that you are respectful and supportive to your fellow students and tutors. Adopting this approach will create a positive atmosphere within sessions and is something you can use in your professional life.

To access some of the rooms and specialist space used for this module you will need your University ID card. Please remember that your University ID should be carried with you always.

Lateness policy

Please, ensure you are on time to sessions as tutors will start sessions promptly.

Mobile phones

All mobile phones must be switched to silent during sessions unless directed by your tutor to do otherwise. Calls and texts cannot be made or received during sessions unless agreed with the tutor prior to the session starting. If you are observed using your mobile phone you can be asked to leave the session.

Academic misconduct

Academic misconduct is a breach of the values of academic integrity, and can occur when a student cheats in an assessment, or attempts to deliberately mislead an examiner that the work presented is their own when it is not. It includes, but is not limited to, plagiarism, commissioning or buying work from a third party or copying the work of others, breach of examination room rules.

Students who attempt to gain unfair advantage over others through academic misconduct will be penalised by sanctions, according to the severity of the offence, which can include exclusion from the University. Links to the relevant University regulations and additional support resources can be found here:

- Academic Integrity Awareness Course. Access to course. (You will have to log into MyUniHub and then MyLearning to access the course)
- Section F: Infringement of Assessment Regulations/Academic Misconduct: `https://www.mdx.ac.uk/about-us/policies/university-regulations`

- Referencing and Plagiarism: Suspected of plagiarism?: http://libguides.mdx.ac.uk/c.php?g=322119&p=2155601

- Referencing and avoiding plagiarism: http://unihub.mdx.ac.uk/your-study/learning-enhancement-team/online-resources/referencing-and-avoiding-plagiarism

- The MDXSU Advice Service offers free and independent support face-to-face in making an appeal, complaint or responding to any allegations of academic or non-academic misconduct. https://www.mdxsu.com/advice

Extenuating circumstances

There may be difficult circumstances in your life that affect your ability to meet an assessment deadline or affect your performance in an assessment. These are known as extenuating circumstances or 'ECs'. Extenuating circumstances are exceptional, seriously adverse and outside of your control. Please see link for further information and guidelines: https://unihub.mdx.ac.uk/your-study/assessment-and-regulations/extenuating-circumstances

7 Module overview and learning schedule

Week	Topic	Recommended reading
1	Options and rational pricing	Chapter 1, Sec. 1.1, 1.2 (Elliott & Kopp, 2004); Chapter 1, 2 (Roman, 2012); Chapter 1, 3 (Crack, 2014)
2	Binomial pricing	Chapter 1, Sec. 1.3–1.5 (Elliott & Kopp, 2004); Chapter 5, 6 (Roman, 2012); Chapter 6, 8, Sec, 8.3.3 (Crack, 2014)
3	Stochastic processes and nowhere differentiable functions	Chapter 6, Sec. 6.1 (Elliott & Kopp, 2004); Chapter 9 (Roman, 2012); Chapter 1, Sec. 3 (Stratonovich, 2014).
4	Gaussian white noise and Wiener process	Chapter 6, Sec. 6.2, 6.4 (Elliott & Kopp, 2004); Chapter 10 (Roman, 2012)
5–6	Continuous Markov process and the diffusion equation	Chapter 7 (Crack, 2014); Chapter 4 (Stratonovich, 2014)
7	Stochastic differential equations and integrals	Chapter 6, Sec. 6.3, 6.5 (Elliott & Kopp, 2004)
8	Ito differentiation rule	Chapter 6, Sec. 6.4 (Elliott & Kopp, 2004)
9–10	The Black-Scholes theory	Chapter 7, Sec. 7.6, 7.9 (Elliott & Kopp, 2004); Chapter 10 (Roman, 2012); Chapter 4, Sec. 4.4, Chapter 8 (Crack, 2014)
11—12	Revision	

8 Assessment

Formative assessment: Formative assessments do not directly contribute to the overall module mark, but they do provide an important opportunity to receive feedback on your learning. They provide an opportunity to evaluate and reflect on your understanding of what you have learnt. They also help your tutors identify what further support and guidance can be given to improve your grade.

On this module, you will receive formative assessment of your homework (problem solutions) during the lectures and feedback to your practical work during the lab sessions.

Summative assessment: Summative assessment is the assessed work that determines the overall module grade. It is the way the University verifies that students have met the learning outcomes at the appropriate level.

There are *two* assessment components in this module: coursework and exam.

9 Coursework

Aims and Objectives

The aim of the coursework is to demonstrate the ability to:

1. Estimate historical volatility of a stock and to price options using the Black-Scholes formulae.
2. Estimate implied volatility using options from a market.
3. Examine the assumptions of the Black-Scholes theory by analysing real stock data.

Your work will be assessed in the classroom and based on a written report, which should include the main results and conclusions.

Software Required

You will need a copy of Microsoft Excel or Open Office Calculator, which will be installed on the University computers. It is advised to back up your work.

Tasks and Assessment Weight

9.0.1 Estimation of historical volatility and Black-Scholes option pricing (30%)

- Download historical quotes for a stock of your choice. You can access such data on:

`http://finance.yahoo.com/`
`http://www.nasdaq.com/symbol/googl/historical`

- Estimate historical drift and volatility from the data. You can find the required information in Roman, 2012, Chapter 10, p. 228 or Crack, 2014, Section 8.7.1, p. 148.

- Compute the values of European put and call options using Black-Scholes formula for a range of stock prices S, and using the average price as the strike price K. You can set arbitrary interest rate (e.g. $r = .05$) and arbitrary expiration time (e.g. $T - t = .5$ years). The required information can be found in Roman, 2012, Chapter 10, p. 236 or Crack, 2014, Section 8.2, p. 122.

- Plot the values of options as functions of stock S.

9.0.2 Estimation of implied volatility (30%)

- Access option chain for your stock. You can view option chains on

  `http://www.nasdaq.com/`
  `http://www.cboe.com/`

- Copy the asking prices for call and put options for different expiration dates $T - t$ and for different strike prices K.

- Estimate implied volatility using the Black-Scholes formula. In MS Excel you can use the `Goal Seek` command to search for the value of volatility that is required to produce a given option price. See Roman, 2012, Chapter 10, pp. 237–239 or Crack, 2014, Section 8.7.2, p. 149 for more information.

- Plot implied volatilities as function of strike price K (these plots are called 'volatility smiles'). Plot such charts for different expiration dates.

9.0.3 Analysis of model assumptions and data (30%)

- You can use your stock data from previous tasks, but you may also wish to download a high frequency (intraday) data for this task. Such data is available on several sites, such as Nasdaq.

- Estimate historical volatility using different intervals from the data, and discuss the assumption that stock is a stationary process.

- Estimate autocorrelation function $k(\tau)$ by computing correlations between stock prices $S(t)$ and $S(t + \tau)$ for a range of $\tau \in [0, T]$.

- Estimate the correlation time τ_{cor} by integrating (summing) the absolute values of the autocorrelation function, and compute the effective frequency. See Stratonovich, 2014, Chapter 2, Section 1, pp. 21–25 for the definitions.

- Discuss you findings in the context of the assumptions about the dynamics of the stock (i.e. δ-correlatedness, Markov property and spectrum of Gaussian white noise).

9.0.4 Presentation (10%)

Your report should be well presented. A good guide is the *Publication Manual* of the American Psychological Association (e.g. see `http://www.apastyle.org/`). At the very least, your report should be clear, typed or nicely hand-written document with good spelling, grammar and easy to understand English. There is no word limit, but a useful report should be just long enough to describe the work. A sensible limit is about 10 pages of typed text. Beyond this, you are probably being a bit too verbose. Tables, graphs, careful labelling and numbering are all well established and effective presentation tools.

Things to avoid are:

- Including images or diagrams that you did not create yourself or did not obtain the permission to use from the author (even if the image is from the Internet).
- Including graphs or diagrams that you do not describe in the text.
- Forgetting to label the axes on the charts.
- Using 3D charts to display 2D information.
- Including material irrelevant to the work.

Assignment Submissions

Submit your report to the Coursework submission link on My Learning or Unihelp office by **Week 11, Friday, 19:00 December 13, 2019**.

10 Exam

- 2 hours written exam.
- You will need to answer 2 questions out of 3.

Feedback on your assignments

You will be provided with feedback on all assessment that is helpful and informative, consistent with aiding the learning and development process.

Feedback will normally be provided within 15 working days of the published assessment component submission date.

Overall module grade

Indicate how the overall module grade will be calculated. For example

Each component of assessment will be marked directly onto the 20-point scale based on the assessment criteria. To produce the overall module grade a weighted average percentage will be calculated using the midpoint percentage in the scale below and then converted to a 20-point grade.

In order to pass this module, you need to pass all assessment tasks with a minimum grade of 16 or equivalent.

Before you submit your work for final grading, please ensure that you have accurately referenced the work. It is your responsibility to check the spelling and grammar. If you have submitted a formative or draft assessment, you will receive feedback but no grade. The comments should inform you about how well you have done or tell you about the areas for improvement. All assignments should be submitted online unless specified in assessment briefs.

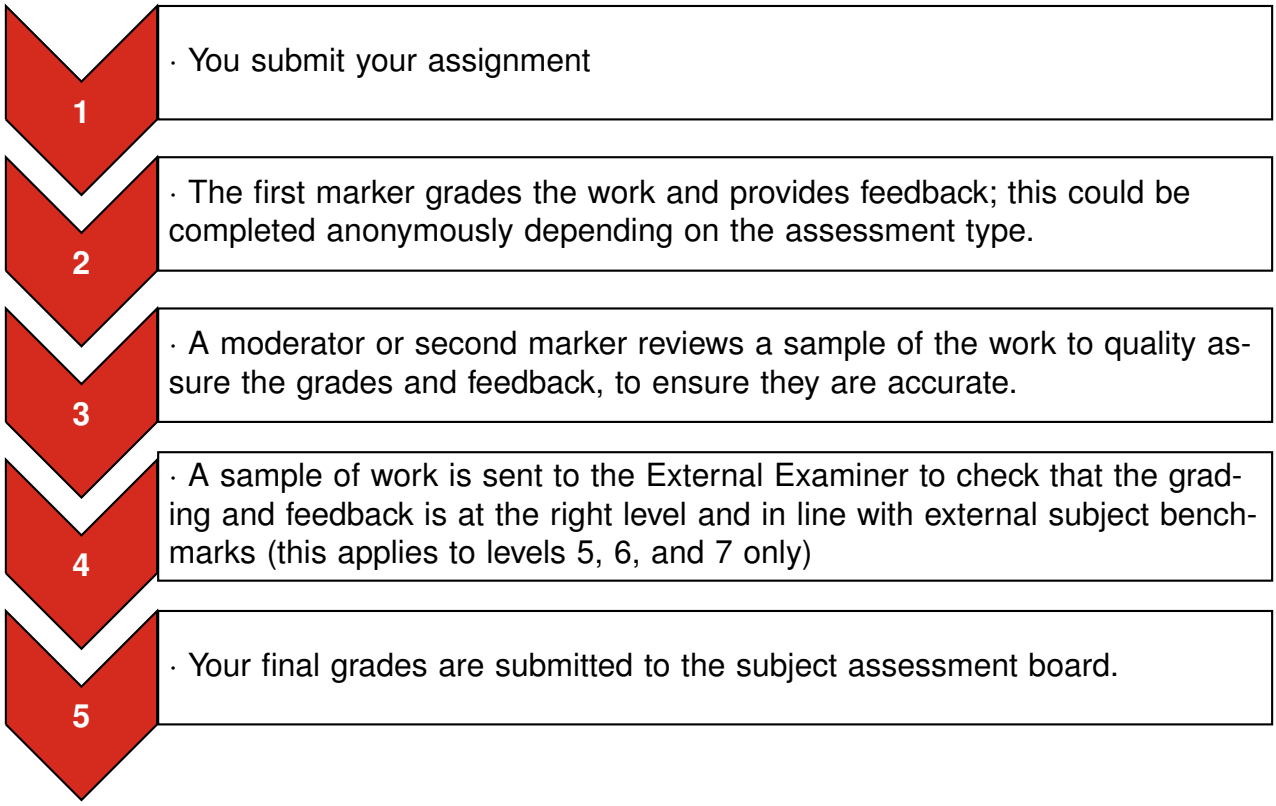
Further information is available at `https://unihub.mdx.ac.uk/study/assessment/regulations`

Classification	Grade	Percentages		Midpoint
Distinction	1	90	100	95.0
	2	80	89	84.5
	3	72	79	75.5
	4	70	71	70.5
Merit	5	68	69	68.5
	6	65	67	66.0
	7	62	64	63.0
	8	60	61	60.5
Pass	9	58	59	58.5
	10	55	57	56.0
	11	52	54	53.0
	12	50	51	50.5
	13	48	49	48.5
	14	45	47	46.0
	15	42	44	43.0
	16	40	41	40.5
Compensatable fail	17	35	39	37.0
	18	30	34	32.0
Uncompensatable fail	19	0	29	14.5

Assessment process

The following diagram provides an overview of the marking process for your module assessment. Details of the programme external examiner can be found in the programme handbook.

Further information on the role of external examiners can be found at. `http://unihub.mdx.ac.uk/your-study/ensuring-quality/external-examiners`



References

Crack, T. F. (2014). *Basic Black-Scholes: Option pricing and trading* (3rd ed.). Timothy Crack.

Elliott, R. J., & Kopp, P. E. (2004). *Mathematics of financial markets* (2nd ed.). Springer.

Roman, S. (2012). *Introduction to the mathematics of finance: Arbitrage and option pricing*. Springer.

Stratonovich, R. L. (2014). *Topics in the theory of random noise* (Vol. 1). Martino Fine Books.