Questions 8

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Question 1

Answer the following questions:

a) What is a fuzzy set?

Answer: *A fuzzy set is a set of elements that have some common property (such as 'Hot'). What makes the set fuzzy is a fuzzy membership function.*

b) What is a membership function of a fuzzy set?

Answer: *Membership function describes the degree of confidence that an object belongs to the set. Usually a fuzzy set is some category (discrete fact, such as 'Hot') related to a real variable (continuous, such as temperature T). Membership function relates the value of the real variable (e.g. a temperature $T = +20°C$) to the fuzzy set by saying how much this particular value belongs to the category (e.g. how much it is true that $T = +20°C$ is 'Hot'). Membership can be measured in percentage from 0% to 100% or as a number from 0 to 1. Sometimes membership function is also called 'confidence factor'. For example, membership $M(Hot)$ for $T = +20°C$ is 80% means that we can be 80% confident that temperature $+20°C$ is hot.*

c) Can a fuzzy membership be `True` and `False` at the same time?

Answer: *Yes. In fact, a fuzzy variable is always* `True` *and* `False` *at the same time, but with different degrees of membership (confidence). Moreover, if M is the membership of a variable in* `True`*, then its membership in* `False` *will be $1 - M$.*

d) What is a fuzzy variable?

> **Answer:** *A collection of fuzzy sets is a fuzzy variable. Usually, the sets of a fuzzy variable are related to the same real variable and describe different categories that can characterise this variable. For example, for a real value temperature the corresponding fuzzy variable can be* {`Cold`, `Hot`}.

Question 2

Consider the following real variables from everyday life:

- Income measured in £UK.
- Speed measured in meters per second.
- A TV show measured in how much you are interested watching it.
- A meal measured in how much you like to eat it.
- A traffic light measured in what colour is on.

In each case, suggest a fuzzy variable corresponding to these real variables. For which of these five variables the use of a fuzzy variable is not really necessary? Why?

Answer: *I suggest the following fuzzy variables (you may come up with a bit different):*

- *Income:* {`Small`, `Medium`, `Large`}
- *Speed:* {`Slow`, `Fast`}
- *A TV show:* {`Boring`, `OK`, `Fascinating`}
- *A meal:* {`Disguisting`, `So − −so`, `Good`, `Delisheous`}
- *A traffic light:* {`Red`, `Yellow`, `Green`}

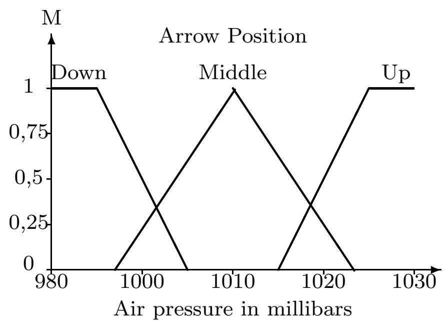
It is not necessary to use the fuzzy representation for a traffic light. The reason for that is that we only have to consider when it is either `Red`, `Yellow` *or* `Green`*, and we do not need to consider intermediate states. Furthermore, it is not really often when you see, say,* `Red` *and* `Green` *at the same time. Thus, fuzzy variables are necessary when we really have to consider 'blurred' states.*

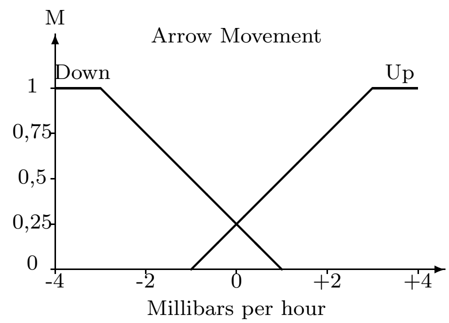
Question 3

Consider the following fuzzy expert system for weather forecast:

Rule	Condition	Action	Confidence
R1:	IF *arrow is down*	THEN *clouds*	$M = 0.8$
R2:	IF *arrow is in the middle* AND *moving down*	THEN *clouds*	$M = 0.6$
R3:	IF *arrow is in the middle* AND *moving up*	THEN *sunny*	$M = 0.6$
R4:	IF *arrow is up*	THEN *sunny*	$M = 0.8$

The following two plots represent the membership functions of two fuzzy variables describing the position of the arrow of barometer (left) and the direction of its movement (right):





The air pressure is measured in millibars, and the speed of its change in millibars per hour. Answer the following questions:

a) How much is the arrow `Down`, `Up` or in the `Middle` if it indicates that the pressure is 1020 millibars? Use membership functions on the graphs.

Answer: *By looking at the left graph (or using a ruler) we can find that when the pressure is 1020 millibars the arrow is up with confidence* $M = 0.5$, *down with* $M = 0$ *and in the middle with* $M = 0.25$.

b) How much is the arrow moving `Down` or `Up` if the pressure changes -2 millibars every hour?

Answer: *Similarly, by looking at the right graph we can say that the arrow is moving down with confidence* $M = 0.75$ *and moving up with* $M = 0$.

c) Using the membership values found above and confidences of the rules in the table calculate the degree of confidence in that the sky is clear or cloudy.

Answer: *The following are the above found degrees of membership of the arrow to five categories:*

$$\begin{aligned} M(\textit{arrow is down}) &= 0 \\ M(\textit{arrow is in the middle}) &= 0.25 \\ M(\textit{arrow is up}) &= 0.5 \\ M(\textit{arrow is moving down}) &= 0.75 \\ M(\textit{arrow is moving up}) &= 0 \end{aligned}$$

First, we find the combined membership values of condition parts of the rules:

$$\begin{aligned} M(\textit{arrow is down}) &= 0 \\ M(\textit{arrow is in the middle } \text{AND } \textit{moving down}) &= \min[0.25, 0.75] = 0.25 \\ M(\textit{arrow is in the middle } \text{AND } \textit{moving up}) &= \min[0.25, 0] = 0 \\ M(\textit{arrow is up}) &= 0.5 \end{aligned}$$

To calculate the degree of confidence in conclusion of each rule, we need to multiply the the membership values of their conditions by the degrees of confidence of the rules itself:

$$M(\textit{action}) = M(\textit{condition}) \times M(\textit{rule}) \; .$$

Using the M values for rules in the table we can calculate

$$\begin{aligned} M_1(\textit{clouds}) &= 0 \times 0.8 = 0 \\ M_2(\textit{clouds}) &= 0.25 \times 0.6 = 0.15 \\ M_1(\textit{sunny}) &= 0 \times 0.6 = 0 \\ M_2(\textit{sunny}) &= 0.5 \times 0.8 = 0.4 \end{aligned}$$

Because we have two conclusions about for each type of weather, we need to use the following formula for combining memberships of two conclusions:

$$M(x) = M_1(x) + M_2(x) - M_1(x) \times M_2(x)$$

So, the results are

$$\begin{aligned} M(\textit{clouds}) &= 0 + 0.15 - 0 \times 0.15 = 0.15 \\ M(\textit{sunny}) &= 0 + 0.4 - 0 \times 0.4 = 0.4 \end{aligned}$$

Question 4

What is the purpose of defuzzyfication? Name at least one method used for defuzzyfication.

Answer: *The process of defuzzyfication is conversion from fuzzy sets in a fuzzy variable into a single real value. The purpose of this is to derive a single crisp conclusion from a set of fuzzy conclusions a system has arrived to (e.g. how much money exactly a bank can let). One common method for defuzzyfication is called* **centroid**. *It uses the 'centre of gravity' of the combined area under of memberships.*

Question 5

Name three strengths and three weaknesses of fuzzy expert systems.

Answer: *Strengths are:*

- *Fuzzy ES need to use fewer rules as opposed to traditional rule–based system. This is because there is no need to cover all the cases.*
- *Because there are fewer rules, it is easier to understand a fuzzy logic expert system.*
- *It is possible to fine tune the system by changing parameters of membership functions. This allows for using a fuzzy system in 'What–If' analysis by strategic planning.*

Weaknesses are:

- *Fuzzy systems usually are more limited in explaining their solutions, than crisp rule–based systems. This is because there are usually fewer rules doing bigger steps in reasoning, and all rules fire in parallel, which may be hard to follow.*
- *They still require the knowledge to be engineered (e.g. by consulting experts).*
- *Saturation problem. If membership functions are not defined carefully (e.g. memberships in fuzzy sets are not well separated), then the system may begin to give the same answer (output value) for all the problems (for different input values). The system in this case will be just generating the average answer all the time.*