Lecture 4:

Feed–Forward Neural Networks

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BIS4435

HISTORICAL BACKGROUND

1943	McCulloch and Pitts proposed the first computational model of a neuron
1949	Hebb proposed the first learning rule
1958	Rosenblatt's work on perceptrons
1969	Minsky and Papert's paper exposed limitations of the theory
1970s	Decade of dormancy for neural networks
1980–90s	Neural network return (self–organisation, back–propagation algorithms, etc)

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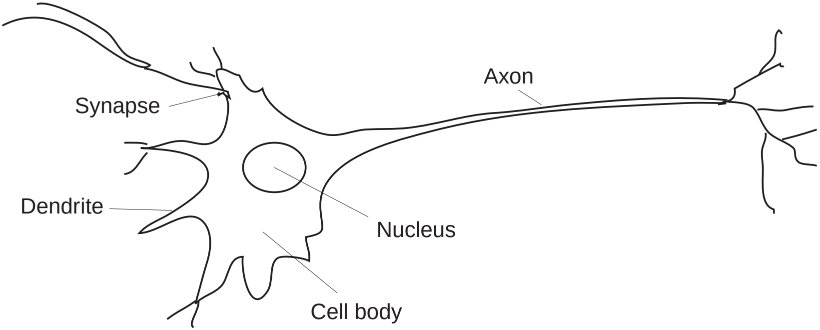
- Each neuron is connected to $\approx 10^4$ others
- Some scientists compared the brain with a 'complex, nonlinear, parallel computer'.
- The largest modern neural networks achieve the complexity comparable to a nervous system of a fly.

NEURONS



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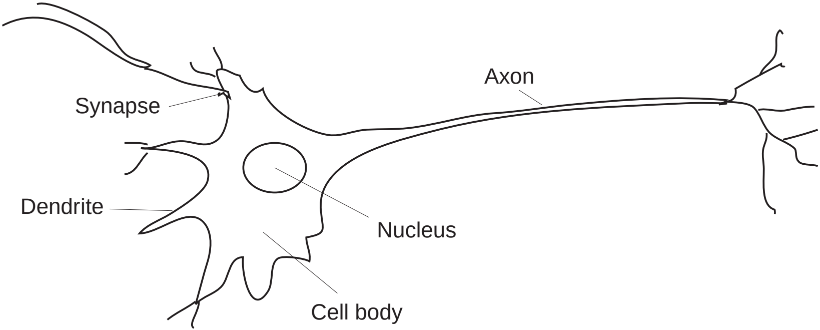

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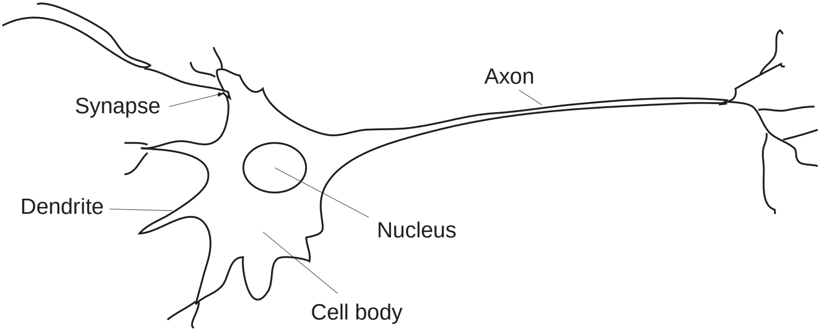

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- The information in transmitted in a form of electro-chemical signals (pulses).
- When a neuron sends the information we say that a neuron '**fires**'.

EXCITATION AND INHIBITION

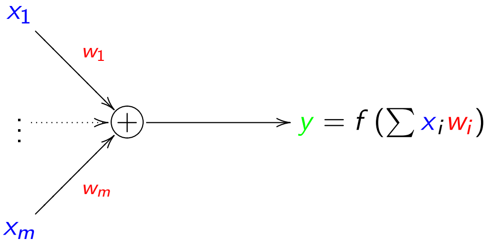
- The receptors of a neuron are called **synapses**, and they are located on many branches, called **dendrites**.
- There are many types of synapses, but roughly they can be divided into two classes:

  Excitatory: a signal received at this synapse ‘encourages’ the neuron to fire

  Inhibitory: a signal received at this synapse inhibits the neuron (as if asking to ‘shut up’)
- The neuron analyses all the signals received at its synapses. If most of them are ‘encouraging’, then the neuron gets ‘excited’ and fires its own message along a single wire, called **axon**.
- The axon may have branches to reach many other neurons.

A MODEL OF A SINGLE NEURON (UNIT)

McCulloch and Pitts (1943) proposed the 'integrate and fire' model:



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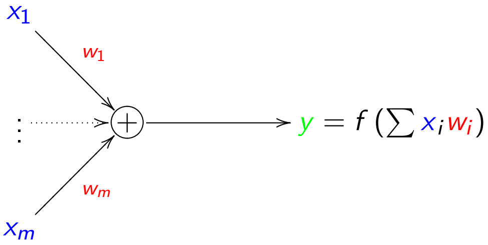

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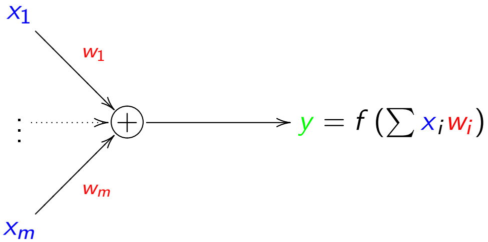

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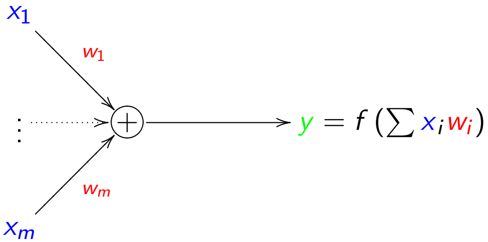

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- The input values are multiplied by their weights and summed

$$v = w_1 x_1 + w_2 x_2 + \cdots + w_m x_m = \sum_{i=1}^{m} w_i x_i$$

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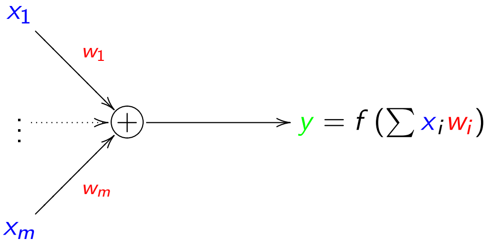

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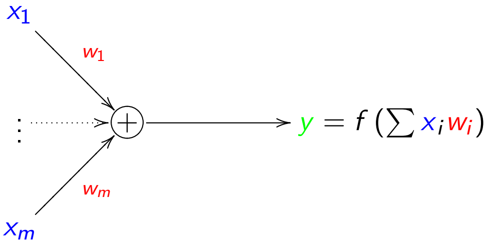

Example

Let $x = (0, 1, 1)$ and $w = (1, -2, 4)$. Then

$$v = 1 \cdot 0 - 2 \cdot 1 + 4 \cdot 1 = 2$$

ACTIVATION FUNCTION

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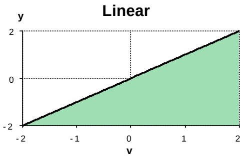
Linear function:

$$f(v) = a + v = a + \sum w_i x_i$$

where parameter a is called **bias**. Noice that in this case, a neuron becomes a linear model with bias a being the *intercept* and the weights, $w_1, \ldots, w_m$, being the *slopes*.



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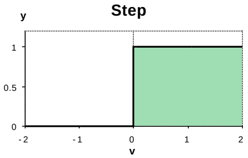
Heviside **step** function:

$$f(v) = \begin{cases} 1 & \text{if } v \geq a \\ 0 & \text{otherwise} \end{cases}$$

Here a is called the **threshold**



Example

If $a = 0$ and $v = 2 > 0$, then $f(2) = 1$, the neuron fires

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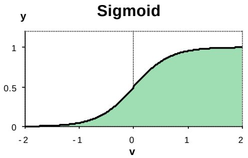
Sigmoid function:

$$f(v) = \frac{1}{1 + e^{-v}}$$



NEURONS AS DATA-DRIVEN MODELS

- We use data to create models representing the relation between the input x and the output y variables (e.g. between income and credit score)

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- If we use data to adjust parameters (the weights) to reduce the error, then a neuron becomes a data-driven model
- If we use only linear activation functions, then a neuron is just a linear model with weights corresponding to slopes (i.e. related to correlations)

$$f(x_1, \ldots, x_m) = {\color{red}a} + {\color{blue}w_1} x_1 + \cdots + {\color{blue}w_m} x_m$$

SO, WHAT IS DIFFERENT FROM LINEAR MODELS?

- The linear mean–square regression is a good technique, but it relies heavily on the use of quadratic cost function

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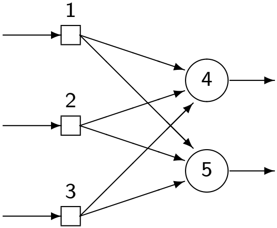
- Networks of many neurons can be seen as sets of multiple and competing models.
- Neural networks can be used to model non-linear relations in data.

FEED-FORWARD NEURAL NETWORKS

A collection of neurons connected together in a network can be represented by a directed graph:



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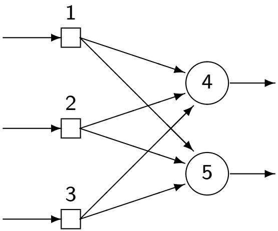

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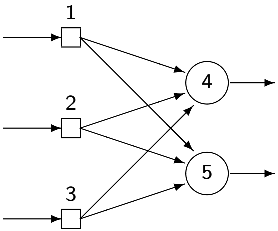

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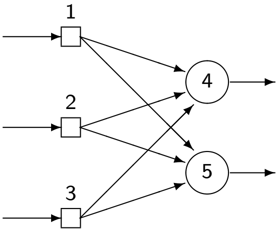
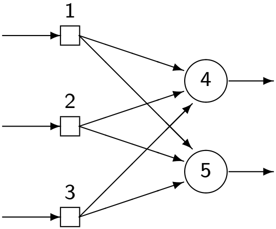


- **Nodes** represent the neurons, and **arrows** represent the links between them.
- Each node has its number, and a link connecting two nodes will have a pair of numbers (e.g. $(1, 4)$ connecting nodes 1 and 4).
- Networks without cycles (feedback loops) are called a feed-forward networks (or **perceptron**).

INPUT AND OUTPUT NODES

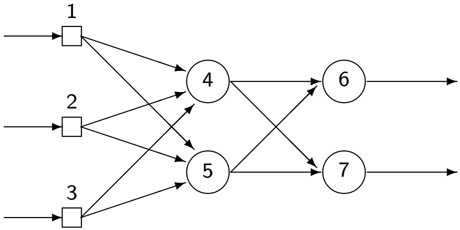


Input nodes of the network (nodes 1, 2 and 3) are associated with the input variables ($x_1, \ldots, x_m$). They do not compute anything, but simply pass the values to the processing nodes.

Output nodes (4 and 5) are associated with the output variables ($y_1, \ldots, y_n$).

HIDDEN NODES AND LAYERS

- A neural network may have hidden nodes — they are not connected directly to the environment ('hidden' inside the network):



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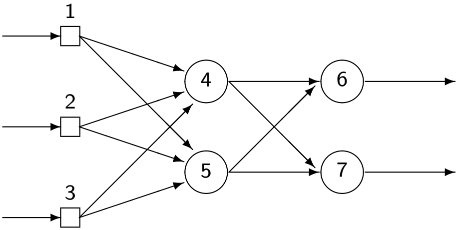

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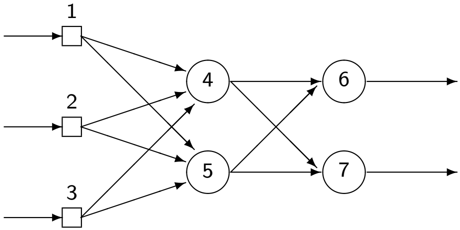

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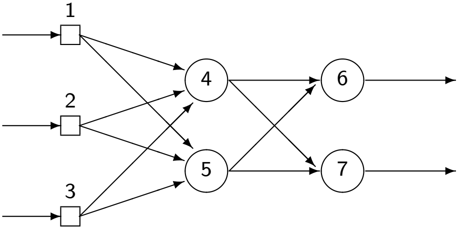
NUMBERING THE WEIGHTS

- Each jth node in a network has a set of weights w_{ij}.



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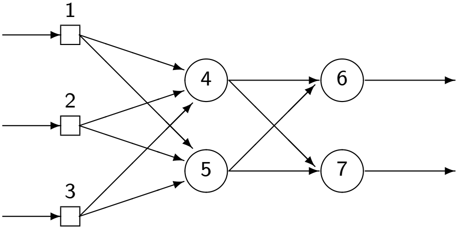
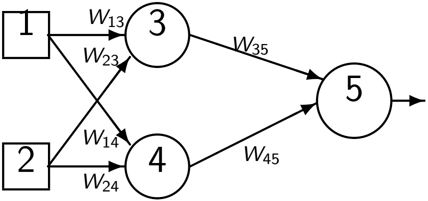


- For example, node 4 has weights w_{14}, w_{24} and w_{34}.
- A network is completely defined if we know its **topology** (its graph), the set of all weights w_{ij} and the activation functions, f, of all the nodes.

Example



$w_{13} = 2$ $w_{23} = -3$	$w_{35} = 2$ $w_{45} = -1$
$w_{14} = 1$ $w_{24} = 4$	

$$f(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

What is the network output, if the inputs are $x_1 = 1$ and $x_2 = 0$?

SOLUTION

1. Calculate weighted sums in the first hidden layer:

$$
\begin{aligned}
v_3 &= w_{13}x_1 + w_{23}x_2 = 2 \cdot 1 - 3 \cdot 0 = 2 \\
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4. The output is $y_5 = f(1) = 1$

TRAINING NEURAL NETWORKS

- Let us invert the previous problem:
  - Suppose that the inputs to the network are $x_1 = 1$ and $x_2 = 0$, and f is a step function.
  - Find values of the weights, w_{ij}, such that the output of the network $y_5 = 0$?

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  - Find values of the weights, w_{ij}, such that the output of the network $y_5 = 0$?
- This problem is more difficult, because there are more unknowns (weights) than knowns (input and output). In general, there is an infinite number of solutions.
- The process of finding a set of weights such that for a given input the network produces the desired output is called training.

SUPERVISED LEARNING

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- We may think of a training set as a set of examples.
- An outline of a supervised learning algorithm:
  1. Initially, set all the weights w_{ij} to some random values
  2. Repeat
     1. Feed the network with an input x from one of the examples in the training set
     2. Compute the network's output $f(x)$
     3. Change the weights w_{ij} of the nodes
  3. Until the error $c(y, f(x))$ is small

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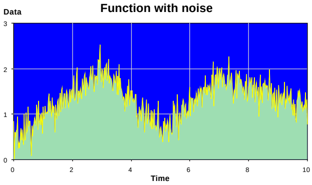
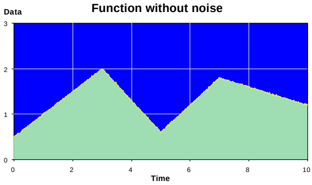
- By changing the weights in the network we may store new information.

GENERALISATION

- By memorising patterns in the data during training, neural networks may produce reasonable answers for input patterns not seen during training (generalisation).





GENERALISATION

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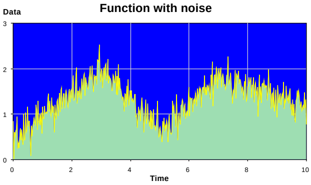
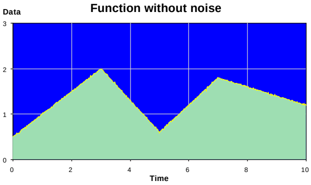
- Generalisation is particularly useful for classification of noisy data, the 'what-if' analysis and prediction (e.g. time-series forecast)





APPLICATION OF ANN

Include:

- Function approximation (modelling)
- Pattern classification (analysis of time–series, customer databases, etc).
- Object recognition (e.g. character recognition)
- Data compression
- Security (credit card fraud)

PATTERN CLASSIFICATION

- In some literature, the set of all input values is called the input pattern, and the set of output values the output pattern

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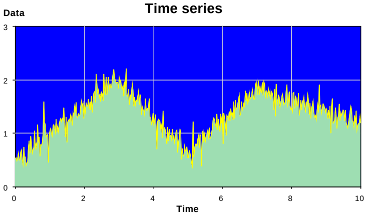
- A neural network 'learns' the relation between different input and output patterns.
- Thus, a neural network performs pattern **classification** or pattern **recognition** (i.e. classifies inputs into output categories).

TIME SERIES ANALYSIS

A time series is a recording of some variable (e.g. a share price, temperature) at different time moments:

$$x(t_1), x(t_2), \ldots, x(t_m)$$



The aim of the analysis is to learn to predict the future values.

TIME SERIES (CONT.)

- We may use a neural network to analyse time series:

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- By training a neural network with m inputs and n outputs on the time series data, we can create such a model.

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- Networks are easy to maintain.

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- Not good for arithmetics and precise calculations.
- Neural networks do not provide explanations. If there are many nodes, then there are too many weights that are difficult to interprete (unlike the slopes in linear models, which can be seen as correlations). In some tasks, explanations are crucial (e.g. air traffic control, medical diagnosis).