Lecture 2:

Uncertainty, Probability and Information

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BIS4435

Historical background

Sources of uncertainty

What is probability?

Joint Probability and Independence

Uncertainty and Information

Reducing the Uncertainty in Data, Random variables

Decision-Making under Uncertainty

HISTORICAL BACKGROUND

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1920-1940	Ronald Fisher, Abraham Wald (statistics)
1948	Claude Shannon (Information theory)

SOURCES OF UNCERTAINTY

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Ignorance : some important information about the system may not be available

Randomness : the system may be random by nature, and thus the uncertainty is irreducible.

WHAT IS PROBABILITY?

Definition

The uncertainty about some event E can range from impossible to certain. Let us denote **probability** of event E by $P(E)$ such that

$P(E) = 0$ means E is impossible;

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Example

For a fair coin, $P(heads) = \frac{1}{2} = 0.5$

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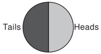
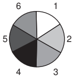
Example

For a fair coin and a fair dice we have

$$\frac{1}{2} + \frac{1}{2} = 1 \qquad\qquad \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 1$$





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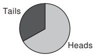

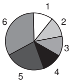

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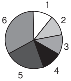
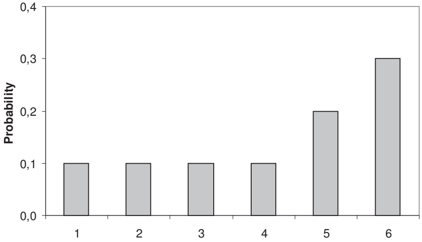


Example

Flip a coin or roll a dice several times to estimate the probabilities.

PROBABILITY DISTRIBUTIONS

We can plot probabilities of all events on a graph, which shows probability **distribution**



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- Do these events occur together simply by chance?
- Probability $P(E_1, E_2)$ is called joint probability of E_1 and E_2.

E_1	E_2
heads	heads
heads	tails
tails	heads
tails	tails

CONDITIONAL PROBABILITY and INDEPENDENCE

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- If E_1 is independent of E_2, then $P(E_1 \mid E_2) = P(E_1)$ and

$$P(E_1, E_2) = P(E_1)P(E_2)$$

SIMPLE CREDIT SCORE EXAMPLE

- You selected at random records of 20 customers and divided them based on *homeowner* and *credit score*.

		Credit score	
		Low	High
Homeowner	No	7	3
	Yes	2	8

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- Can you tell from this data whether credit score and homeownership depend on each other?

SIMPLE CREDIT SCORE EXAMPLE (sol.)

		Credit score		
		Low	High	
Homeowner	No	7	3	**10**
	Yes	2	8	**10**
		9	**11**	**20**

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Our simple test for independence $P(\mathrm{H},\mathrm{C}) \stackrel{?}{=} P(\mathrm{H})P(\mathrm{C})$

UNCERTAINTY and INFORMATION

- If each event can have 2 states (i.e. True, False), then for a system that consists of H number of events there are

$$M = 2^H \text{ possible states}$$

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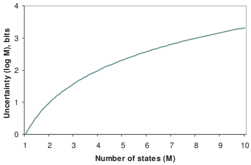
- To measure uncertainty we can use $H = \log_2 M$ (aka *entropy*)



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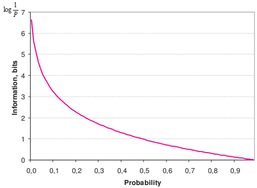

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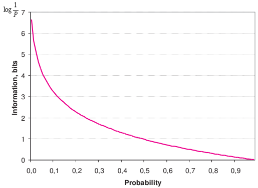

Example

Compare information from observing events with probabilities $\frac{1}{8}$ and $\frac{1}{2}$

REDUCING THE UNCERTAINTY IN DATA

Case:	Age	Gender	M. Income ($ K)	M. Expenses ($ K)	Home owner	Credit score
1	21	0	2	1	0	3
2	18	1	1	2	0	1
3	50	1	6	2	1	5
4	23	0	3	1	1	4
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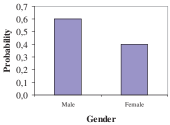
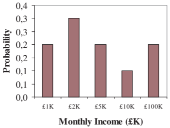
Random variables if each value is associated with probability

$$P(\text{Male}) = \frac{3}{5}, \quad P(\text{Female}) = \frac{2}{5}, \quad (P(\text{Male}) + P(\text{Female}) = 1)$$

IS THERE STRUCTURE IN DATA?

- Using the concept of random variables, we can analyse the distributions of each variable in the database





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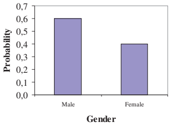

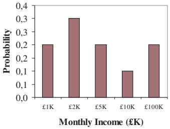

- Each case in the database can be seen as a complex (joint) event (e.g. Case 1 is Age=21, Gender=Female, etc).
- Thus, the whole database can be seen as a joint probability

$$P(\text{Case}) = P(\text{Age, Gender, Income, Expenses, H. owner, C. score})$$

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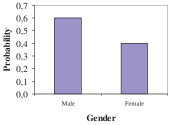

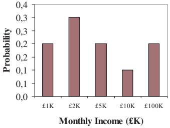

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- Are these variables independent or not?

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- If variable x has n possible values X_1, X_2,...,X_n with probabilities $P(X_1)$, $P(X_2)$,...,$P(X_n)$, then we can compute the expected value

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- If all $P(x) = \frac{1}{n}$, then $E\{x\}$ is simply the average (the mean) value.

MEASURES of LOCATION (cont.)

Example

Variable Age has values 21, 18, 50, 23, 40. Each values occurs once, therefore $P(x) = \frac{1}{5}$ and the expected value is

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$$E\{\text{Age}\} = \frac{21 + 18 + 50 + 23 + 40}{5} = 30,4$$

which is also the average Age.

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- For our data, this is the expected case (i.e the average case)

$$E\{\text{Case}\} = (E\{\text{Age}\}, E\{\text{Gender}\}, \ldots, E\{\text{C. score}\})$$

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- Standard deviation is $Sdev\{x\} = \sqrt{Var\{x\}}$

MEASURES of DISPERSION (cont.)

Example

Find $Var\{\text{Age}\}$ and $Sdev\{\text{Age}\}$?

1. Earlier we found $E\{\text{Age}\} = 30,4$.

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3. Then we multiply each by $P(\text{Age}) = \frac{1}{5}$, and their sum gives the variance

$$Var\{\text{Age}\} = \frac{1}{5}((21-30,4)^2 + \cdots + (40-30,4)^2) = 154,64$$

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$$Var\{\text{Age}\} = \frac{1}{5}((21 - 30,4)^2 + \cdots + (40 - 30,4)^2) = 154,64$$

4. Standard deviation is a square root of the variance

$$Sdev(Age) = \sqrt{154,64} = 12,44$$

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- Note that $Cov(x, y) = Cov(y, x)$ and $Cov(x, x) = Var\{x\}$

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- Note that $Cov(x, y) = Cov(y, x)$ and $Cov(x, x) = Var\{x\}$
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- If x and y are not 'similar', then $Cov(x, y) \approx 0$

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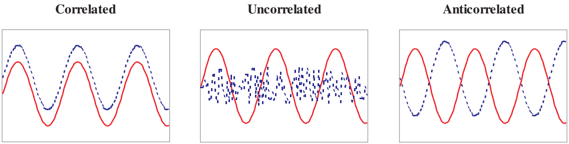

$Corr(x,y) = 1$ $\qquad$ $Corr(x,y) = 0$ $\qquad$ $Corr(x,y) = -1$

CORRELATION MATRIX

- Correlations (or covariances) can tell us how ‘similar’ are two random variables.

	Age	Gender	Income	Expenses	H. owner	C. score
Age	1,0	0,6	0,9	0,6	0,4	0,5
Gender	0,6	1,0	0,2	1,0	-0,2	-0,3
Income	0,9	0,2	1,0	0,2	0,7	0,9
Expenses	0,6	1,0	0,2	1,0	-0,2	-0,3
H. owner	0,4	-0,2	0,7	-0,2	1,0	0,9
C. score	0,5	-0,3	0,9	-0,3	0,9	1,0

CORRELATION MATRIX

- Correlations (or covariances) can tell us how 'similar' are two random variables.
- Below is the correlation matrix showing correlations of each pair of variables in our database

	Age	Gender	Income	Expenses	H. owner	C. score
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- There are many ways to estimate (predict) using probabilities. The most popular are using the expected value and the maximum likelihood (the most probable value).

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- If d_1 and d_2 are two alternative decisions, then we choose d_1 if

$$E\{u \mid d_1\} \geq E\{u \mid d_2\}$$

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$$E\{u \mid A\} = \$1000\frac{1}{2} - \$100\frac{1}{2} = \$450$$

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Thus, we prefer A to B.

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- The analysis of probabilities can show that some events or variables are not independent, and therefore the data is not entirely random (i.e. the uncertainty is not so great).
- For businesses, the information from their corporate data can lead to a discovery of new knowledge.
- This process is often called *knowledge discovery in databases*