Lecture 7: Formal Concept Analysis

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1 Introduction to Concepts

Concepts

Definition 1 (Knowledge). Justified true belief. Can be justified by facts (Nonaka & Takeuchi, 1995).

- Beliefs and propositions are described in terms of concepts or ideas related to each other.
- Concepts represent classes or categories or objects.

Definition 2 (Concept). describes a set of objects or instances which occur in the application domain and which share similar characteristics. (Schreiber et al., 1999, p. 92)

Example 3. Space, planets, society, country, transport, education, knowledge

Extent and Intent of a Concept

Definition 4 (Extent). is the set of all objects that belong to a concept.

Definition 5 (Intent). is the set of all attributes shared by the objects in a concept.

- A concept is determined by its extent and intent.
• It is usually impossible to list all the objects or name all the attributes of a concept.

• Fixing a context is fixing the set of objects and attributes.

Definition 6 (Context). consists of set \( G \) of objects, set \( M \) of attributes and a correspondence (a cross-table) relating objects with values of the attributes.

Example: A Context for Planets

<table>
<thead>
<tr>
<th></th>
<th>size</th>
<th>dist. from sun</th>
<th>moon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>small</td>
<td>near</td>
<td>far</td>
</tr>
<tr>
<td>Mercury</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Venus</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Earth</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Mars</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Jupiter</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Saturn</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Uranus</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Neptune</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Pluto</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

Objects: planets (Mercury, Venus, Earth,...)

Attributes: properties relating to size, distance, satellite.

Example 7. 1. Consider the set of all attributes of Earth: \( B = \{ \text{Small, Near, Yes} \} \)

2. Consider the set of all similar objects: \( A = \{ \text{Earth, Mars} \} \)

3. Then the pair \((A, B)\) is a concept in this context.

2 Ordering of Concepts

Ordering of Concepts by Abstraction

Question 1. • Construct a concept starting from Venus.

• Construct a concept starting from Moon=No.

• Construct a concept starting from Earth and Pluto

• Concept \((A_1, B_1)\) is more general than \((A_2, B_2)\) if and only if the extent \( A_1 \) contains \( A_2 \):

\[
(A_1, B_1) \geq (A_2, B_2) \iff A_1 \supseteq A_2
\]

• This is equivalent to the intent \( B_1 \) contained in \( B_2 \):

\[
(A_1, B_1) \geq (A_2, B_2) \iff B_1 \subseteq B_2
\]

Example 8. • \( A_1 = \{ \text{Earth, Mars, Pluto} \}, B_1 = \{ \text{Small, Yes} \} \)

• \( A_2 = \{ \text{Earth, Mars} \}, B_2 = \{ \text{Small, Near, Yes} \} \)
Taxonomy

- Taxonomy originated in biology as classification of species, their organisation into a hierarchical classification.
- Non-biological taxonomy is classification of objects into categories and subcategories.

Example 9. A power set $2^A$ of set $A = \{a, b, c\}$:

```
\{a, b, c\}  \\
/     \  \   \\
\{a, b\} \{a, c\} \{b, c\}  \\
/     \      \    \\
\{a\} \{b\} \{c\}  \\
/     \  \   \\
∅      \\
```

Concept Lattice

- Concepts can be categorised and organised into a concept lattice.
- The concept lattice visually depicts the relation between concepts.

Example 10 (Concept lattice for planets).

```
small  yes  far  \\
/     \  \   \\
near \ {M,V} \ {E,M} \ {P} \ {J,S} \ {U,N}  \\
/     \      \    \\
no \    \    \    \\
∅      \\
```

3 Concept Analysis Technique

Concept Analysis Techniques
• It is highly recommended to undertake concept analysis exercise in an organisation (Dalkir, 2011).

• Concept analysis involves three dimensions of a given concept:
  1. A list of key attributes.
  2. A list of illustrative examples.
  3. A list of illustrative non-examples.

<table>
<thead>
<tr>
<th>Concept Name</th>
<th>Key Attributes</th>
<th>Examples</th>
<th>Non-examples</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1...</td>
<td>1...</td>
<td>1...</td>
</tr>
<tr>
<td></td>
<td>2...</td>
<td>2...</td>
<td>2...</td>
</tr>
<tr>
<td></td>
<td>3...</td>
<td>3...</td>
<td>3...</td>
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<td></td>
<td>...</td>
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<td>...</td>
</tr>
</tbody>
</table>

Automatic Formal Concept Analysis

• Formal Concept Analysis (FCA) can be performed automatically by FCA algorithms in computer programs.

• It can be used not only for a basic concept analysis, but also as a data-mining technique on large datasets.

• Examples of FCA applications:
  1. FCA for MS Excel http://www.fca.radvansky.net/
  2. Galicia http://www.iro.umontreal.ca/~galicia/
  5. FCA demo http://www.upriss.org.uk/fca/fcademo.html

Summary

• Concepts represent categories or classes of objects (extent) sharing common attributes (intent).

• Concepts can be organised and classified into a hierarchy (taxonomy).

• A concept lattice is based on ordering of concepts by generality.

• Automatic methods based on Formal Concept Analysis can help in the process of organising and representation of knowledge.

Additional Reading

• Read the article ‘Formal conceptualisation as a basis for a more procedural knowledge management’ by Andradea et al. (2007).
References


