

Questions 3: Logic and Rule-Based Reasoning

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Question 1

Use the equivalence of Boolean operations and set-theoretic operations to prove the duality (De Morgan's) laws:

$$\neg(a \wedge b) = \neg a \vee \neg b, \quad \neg(a \vee b) = \neg a \wedge \neg b$$

Hint: you can use Venn diagrams.

Answer: Use the following correspondence:

$\neg a$ — set compliment $\bar{A} = U - A$, where U is a universal set and $A \subset U$.

$a \wedge b$ — set intersection $A \cap B$.

$a \vee b$ — set union $A \cup B$.

Question 2

Boolean variables take values in set $\{0, 1\}$, and elements 0 and 1 satisfy the law of contradiction

$$a \wedge \neg a = 0$$

and the law of excluded middle

$$a \vee \neg a = 1$$

Consider three sets $\emptyset \subset A \subset U$, where \emptyset is the empty and U is the universal set. Which of these sets correspond to 0 and 1 in Boolean logic? Use set-theoretic operations and the above laws to justify your answer.

Answer: \emptyset corresponds to 0, and U to 1.

Question 3

Consider the following production rule:

IF *green* THEN *walk*

a) What is the antecedent of this rule?

Answer: *Antecedent is the left-hand-side of the rule: IF green.*

b) What is the consequent of this rule?

Answer: *Consequent is the right-hand-side of the rule: THEN walk.*

c) Which part of the rule will be matched against the working during the recognise-act cycle?

Answer: *Left-hand-side*

Question 4

The following is the rule set of a simple weather forecast expert system:

1	IF	<i>cyclone</i>	THEN	<i>clouds</i>
2	IF	<i>anticyclone</i>	THEN	<i>clear sky</i>
3	IF	<i>pressure is low</i>	THEN	<i>cyclone</i>
4	IF	<i>pressure is high</i>	THEN	<i>anticyclone</i>
5	IF	<i>arrow is down</i>	THEN	<i>pressure is low</i>
6	IF	<i>arrow is up</i>	THEN	<i>pressure is high</i>

a) Apply these rules if the working memory contains the fact: *arrow is down*. Show your answer in a table listing the rules matching the working memory (conflict set), which rule you apply, and changes to the working memory contents:

Cycle	Working Memory	Conflict set	Rule fired
⋮	⋮	⋮	⋮

Answer:

Cycle	Working Memory	Conflict set	Rule fired
0	arrow is down	5	5
1	arrow is down, pressure is low	5, 3	3
2	arrow is down, pressure is low, cyclone	5, 3, 1	1
3	arrow is down, pressure is low, cyclone, clouds	5, 3, 1	Halt

b) Suppose that the user interface of our ES allows the system to ask a user about the facts whether they are true or false. What question (or questions) the system should ask the user in order to conclude that the sky is clear? What will the user answer? Which rule will require the clarification from the user?

Answer: *The system should ask something like:*

Is the arrow up? (Yes/No)

And the user will answer 'Yes'. The system will start from the goal clear sky, and then, in order to prove it, the system will reason backwards using rules 2, 4 and 6 until it will need to clarify the condition of rule 6. That is whether the fact arrow is up is true or false. Thus, rule 6 will cause the clarification from a user.