Questions 1:
Sets and Mappings

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Question 1

Consider a set of all integers $z \in \mathbb{Z}$ such that $z^2 < 10$. Write this set in a set comprehension notation. Is this a finite set? What happens if the condition is changed to $z^3 < 10$?

Answer: *Set comprehension is* $X = \{z \in \mathbb{Z} : z^2 < 10\}$. *This set can also be written explicitly as* $X = \{-3, -2, -1, 0, 1, 2, 3\}$, *and so it is finite.*

If $X = \{z \in \mathbb{Z} : n^3 < 10\}$, *then the set will include all integers* $z \leq 2$, *because* $z^3 \leq 0$ *if* $z \leq 0$. *Hence the set is infinite.*

Question 2

Let $X = \{a, b, c, d, e\}$ and $Y = \{1, 2, 3, 4, 5\}$. Consider the following correspondences $R \subseteq X \times Y$:

$$\begin{aligned} f &: \{(a,1),(a,2),(a,3),(b,4),(c,5)\} \\ g &: \{(a,4),(b,4),(c,4),(d,4),(e,4)\} \\ h &: \{(a,5),(b,4),(c,3),(d,2),(e,1)\} \end{aligned}$$

Which of the above are functions (mappings)? Which functions are surjective (onto), injective (one-to-one) or bijective (one-to-one correspondence)? Which has an inverse function?

Answer: f *is not a function because: a) It is not defined for all elements of* X *(i.e. no relation for* $d, e \in X$*); b) There is an element in* $a \in X$ *related to more than one elements of* Y *(i.e. to* $1, 2, 3 \in Y$*).*

g *is a function (defined for all elements of* X*, and each* $x \in X$ *is related to precisely one element of* Y*). It is not surjective because* X *are not mapped to all elements of* Y *(in fact, only one element* $4 \in Y$ *is used). It is not injective because there are different elements in* X *corresponding to the same element of* Y *(e.g. both* a *and* $b \in X$ *correspond to* $4 \in Y$*). Thus, the inverse of* g *is not a function.*

h is a function. It is both surjective and injective, and hence it is bijective. Therefore, its inverse $h^{-1} : Y \to X$ is a function (also bijective).