

Questions 6: Uncertainty and Information

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Question 1

Name and briefly describe three main sources of uncertainty.

Question 2

What is probability? What is the probability of ‘raining or not raining’ tomorrow?

Question 3

Suppose that a university database has a variable called ‘Age’ storing the age of a student, and it can have 100 values. What is your estimate of the prior probability of each value? What if the database contains records of a 20,000 of students, and all of them are older than 16?

Question 4

The probability of disjunction (logical OR) of several disjoint events is simply the sum of their probabilities. For example, for two independent events E_1 and E_2

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2)$$

If, however, the events are not disjoint (i.e. the events can happen together), then

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

where $P(E_1 \cap E_2)$ is the joint probability of E_1 and E_2 (logical AND).

- a) What is the probability of disjunction of two events that are not disjoint but are independent of each other?
- b) Consider two fair and independent coins tossed together. What is the probability that at least one of the coins will be ‘Heads’ (i.e. $P(\text{heads OR heads})$)?

Question 5

Suppose you know $P(A \cap B)$ - the joint probability distribution of events A and B . Let also $P(A)$ and $P(B)$ be the probabilities of each event individually (i.e. the marginal probabilities, which can be computed from the joint distribution).

- a) What these probabilities can tell you about the relation between events A and B ?
- b) How could you use this information to reduce the uncertainty about one event based on information about another?
- c) Why is it better to use the information about event A to assess the probability of B , then simply using probability $P(B)$?

Question 6

Consider two systems: A bicycle and an airplane. Why is the uncertainty associated with an airplane higher than the uncertainty of a bicycle?

Question 7

Suppose that a database has recorded a very unusual case — its values are very different from other, more typical cases. Why is this case more interesting from the information theory point of view?