The Use of Entropy for Analysis and Control of Cognitive Models

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Cognitive Psychology

- Ss + performance
- Learning

$\longrightarrow ? \longleftarrow$

Information Theory

- Uncertainty + Information
- Learning

Cognitive Modelling

$\downarrow$

Cognitive Psychology

- Ss + performance
- Learning

$\longrightarrow + \longleftarrow$

Information Theory

- Uncertainty + Information
- Learning

OVERVIEW

- Definitions and calculations of entropy in a cognitive architecture
- Applied to a model
- Use of entropy for analysis
- Use of entropy for control

ENTROPY

Canonical entropy (maximum)	$H_{\max} = \ln M$
Hartley entropy (per random state)	$H_\xi = -\ln P(\xi)$
Boltzmann entropy (Shannon)	$H = E\{H_\xi\} = -\sum_\xi P(\xi) \ln P(\xi)$
Shannon information	$I_{xy} = H_x - H_{x\|y}$

ENTROPY OF SUCCESS

Consider a problem solver with a goal. Let us define two states with respect to achieving the goal:

$$\xi = \begin{cases} \textbf{Success} & \text{if the goal is achieved} \\ \textbf{Failure} & \text{otherwise} \end{cases}$$

$$H_{\mathbf{FS}} = -\left[P(\mathbf{F}) \ln P(\mathbf{F}) + P(\mathbf{S}) \ln P(\mathbf{S})\right] ,$$

Note that $P(\mathbf{F}) = 1 - P(\mathbf{S})$.

PROBABILITY OF SUCCESS

Suppose that to achieve the goal we have a set of n alternative decisions. If the success depends on decisions, then

$$P(\mathbf{S}) = \sum_{i=1}^{n} P(\mathbf{S}, i) = \sum_{i=1}^{n} P(\mathbf{S} \mid i) P(i) \,,$$

where is are the set of alternatives.

- How to calculate $P(\mathbf{S} \mid i)$ and $P(i)$ (and then $H_{\mathbf{FS}}$)?
- This depends on the architecture you use (e.g. ACT–R, SOAR, ANN, etc).

ACT–R COGNITIVE ARCHITECTURE

In ACT–R (Anderson & Lebiere, 1998) each alternative i is represented by a production rule in a conflict set. A rule that wins should have the highest *utility*:

$$\boxed{U_i = P_i G - C_i + \text{noise}(s)}$$

rule's properties :

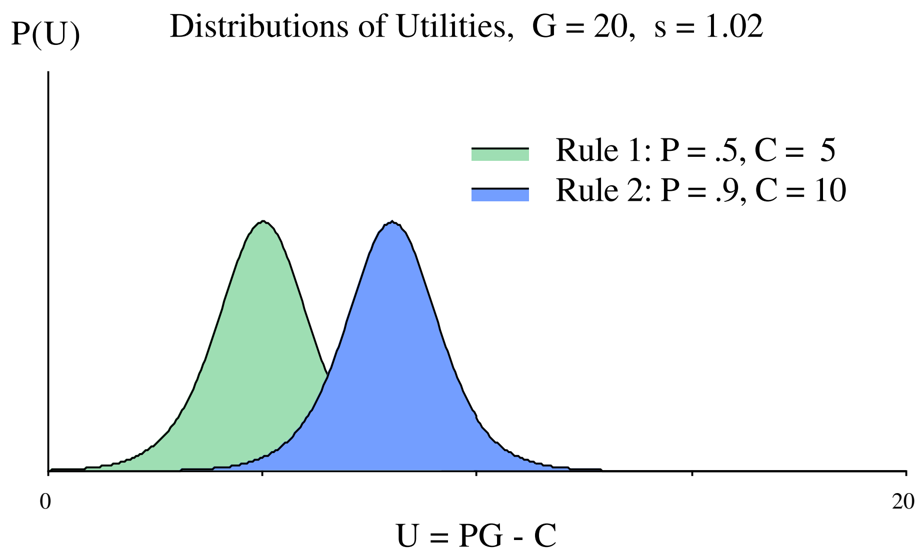
P_i – probability

C_i – cost (e.g. time)

global parameters :

G – goal value (in time units)

s – controls the noise variance



ACT–R COGNITIVE ARCHITECTURE

In ACT–R each rule i has empirical probability calculated from the rule's statistics:

$$P(\mathbf{S} \mid i) \approx P_i = \frac{\text{Successes}_i}{\text{Successes}_i + \text{Failures}_i}\,.$$

If we do not take into account other mechanisms in ACT–R, then

$$P(i) = \frac{e^{\bar{U}_i/\tau}}{\sum_j e^{\bar{U}_j/\tau}}\,.$$

(Boltzmann 'soft–max' equation). Here τ is called the *noise temperature* ($\tau^2 = 6\sigma^2/\pi^2 = 2s^2$).

ENTROPY OF SUCCESS IN ACT–R

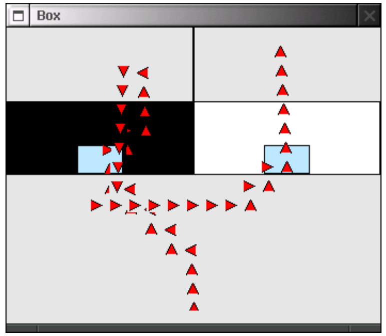
$$P(\mathbf{S}) \approx \sum_{i=1}^{n} P_i \frac{e^{\bar{U}_i/\tau}}{\sum_{j=1}^{n} e^{\bar{U}_j/\tau}}$$

$$P(\mathbf{F}) = 1 - P(\mathbf{S})$$

$$H_{\mathbf{FS}} = -\left[P(\mathbf{F}) \ln P(\mathbf{F}) + P(\mathbf{S}) \ln P(\mathbf{S})\right]$$

A MODEL EXAMPLE

A modified ACT–R 4 model (Belavkin, 2001) of the ‘dancing mouse’ experiment (Yerkes & Dodson, 1908, ~ 700 citations)



Choose1st:

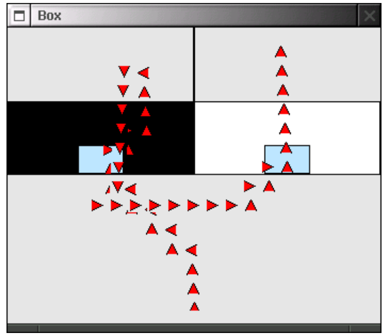
IF	the goal is a *choice* of *first* or *second*
THEN	focus on *first*

Choose2nd:

IF	the goal is a *choice* of *first* or *second*
THEN	focus on *second*

A MODEL EXAMPLE

A modified ACT–R 4 model (Belavkin, 2001) of the ‘dancing mouse’ experiment (Yerkes & Dodson, 1908, ~ 700 citations)



Choose1st:

IF the goal is a *choice* of *first* or *second*

THEN focus on *first*

Choose2nd:

IF the goal is a *choice* of *first* or *second*

THEN focus on *second*

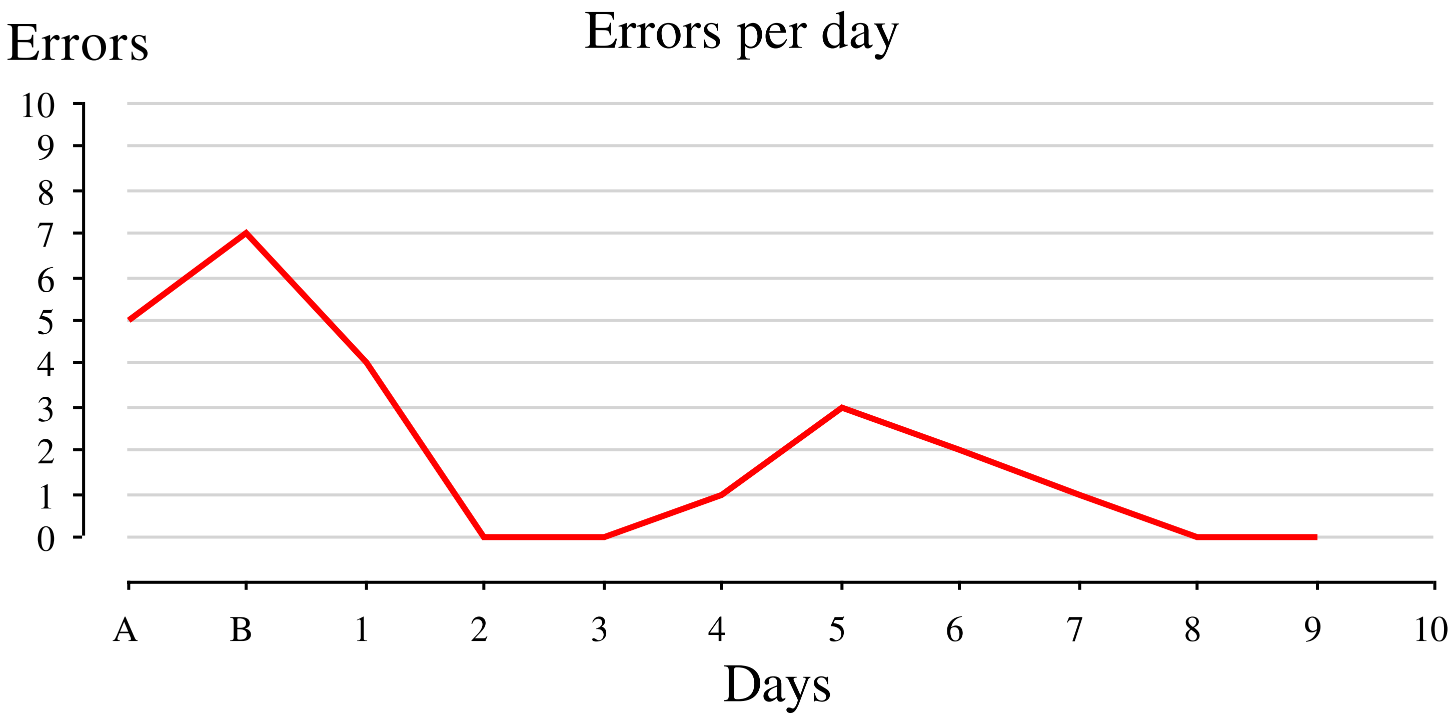
New-Choice-Rule:

IF the goal is a *choice* of *first* or *second*

AND *first* is a *black* door

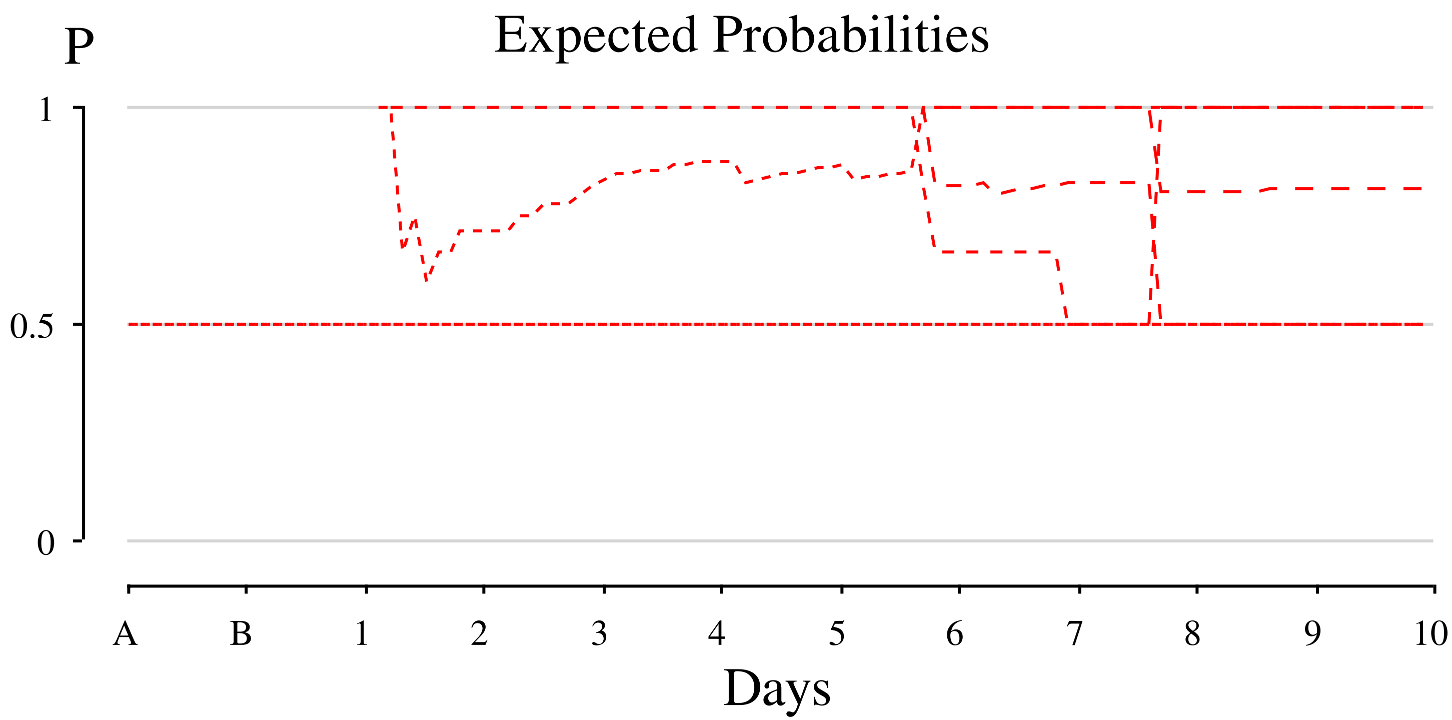
THEN focus on *second*

AN EXAMPLE LEARNING CURVE



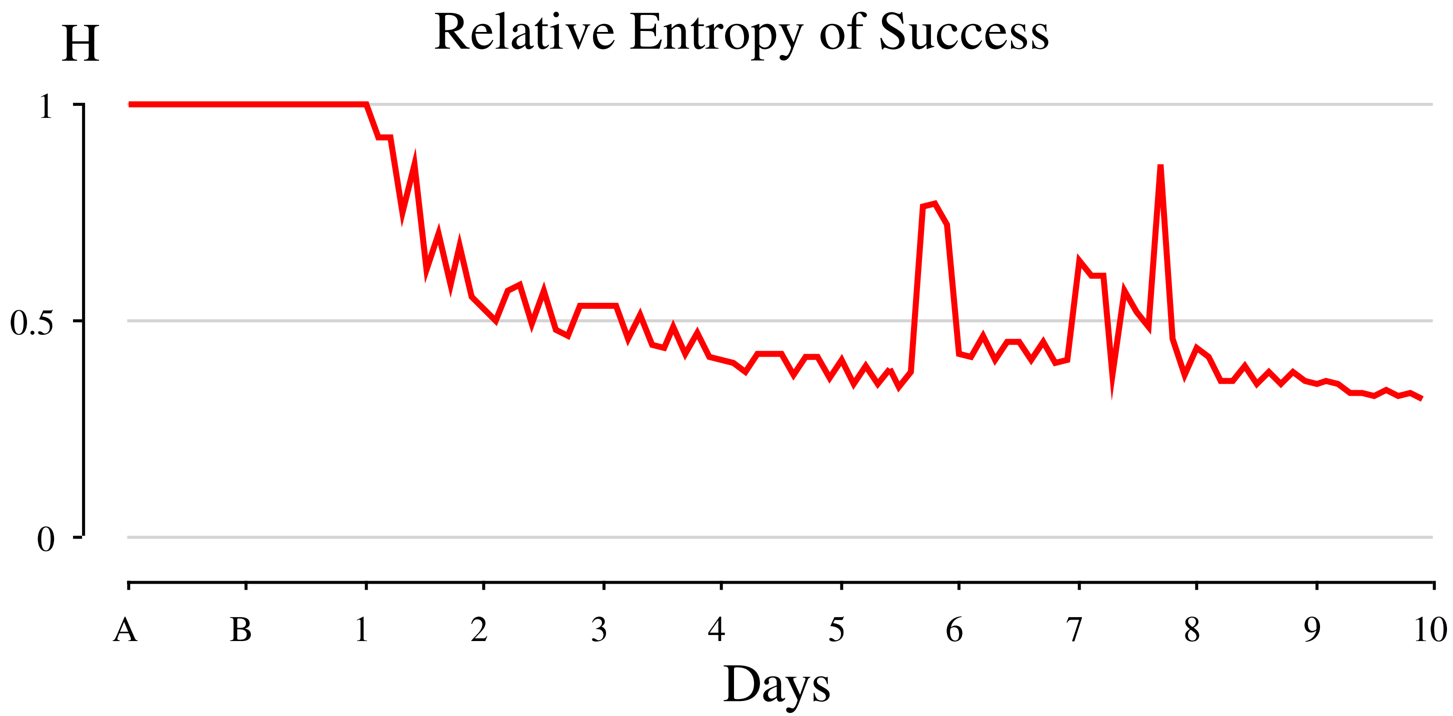
Performance improves, but what and when is learned?

PROBABILITY TRACES



Provide more information about the learning.

DYNAMICS OF ENTROPY



$$H_{\text{rel}} = \frac{H_{\textbf{FS}}}{H_{\max}}, \quad \text{where } H_{\max} = \ln 2$$

ESTIMATING THE KNOWLEDGE

$$P(\mathbf{S}) \approx \sum_{i=1}^{n} P_i \, \frac{e^{\bar{U}_i/\tau}}{\sum_{j=1}^{n} e^{\bar{U}_j/\tau}}$$

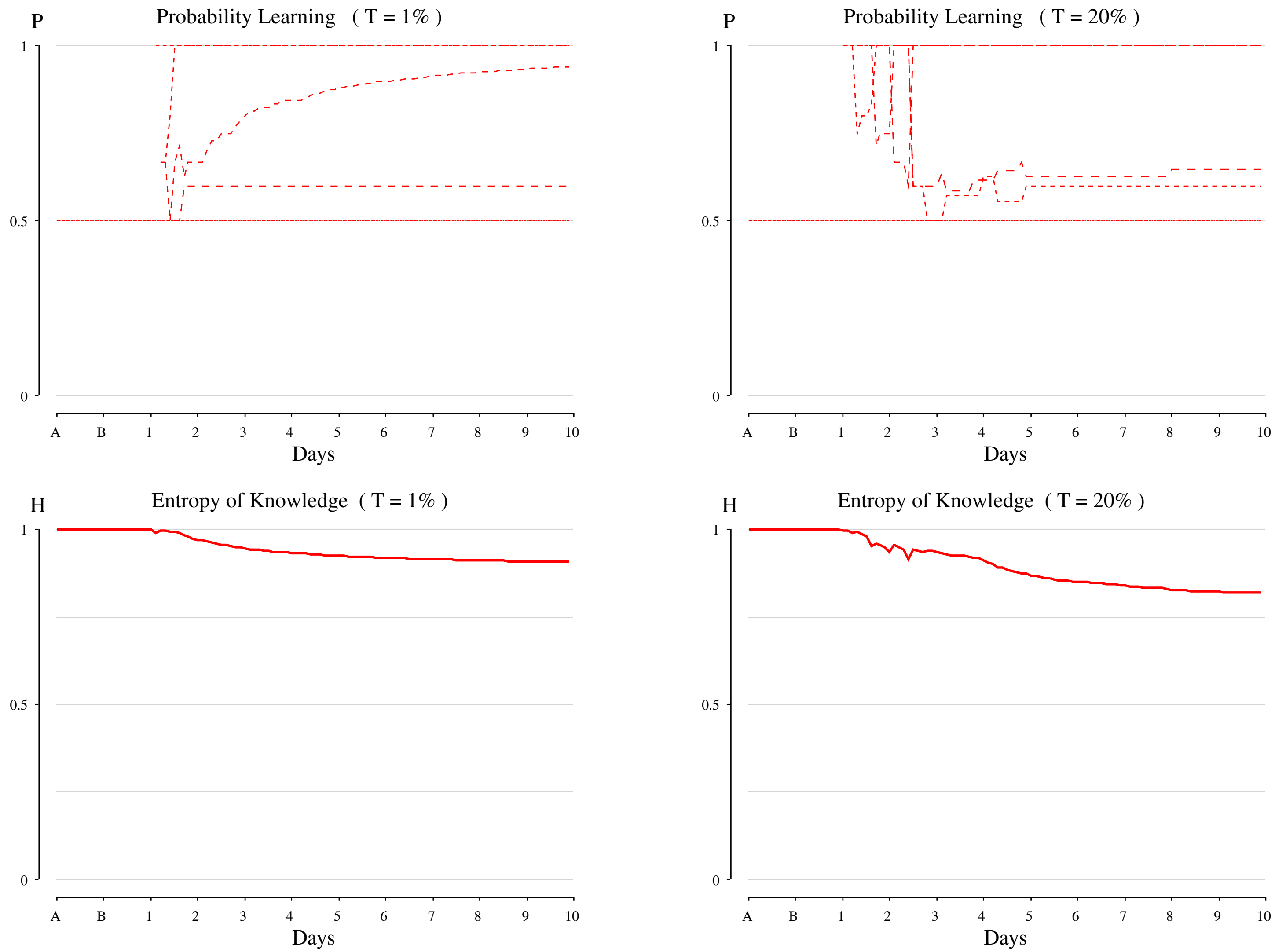
How about different noise settings (τ)?

Let us calculate the entropy independent of the decision making mechanism by assuming

$$P(i) = \frac{1}{n} \qquad \Rightarrow \qquad P(\mathbf{S}) = \sum_{i=1}^{n} P_i \, \frac{1}{n} \, .$$

The entropy H_{k} (entropy of knowledge) can be used to measure the speed of learning for different noise settings.

NOISE FACILITATES LEARNING



ADAPTIVE NOISE

Changes of noise (randomness in behaviour) can optimise the learning of a model:

1. High noise in the beginning of problem exploration allows gaining information more quickly.
2. After learning the important information (rules), keeping the noise low should improve the performance.
3. Noise increase if the environment changes (the number of errors increases).

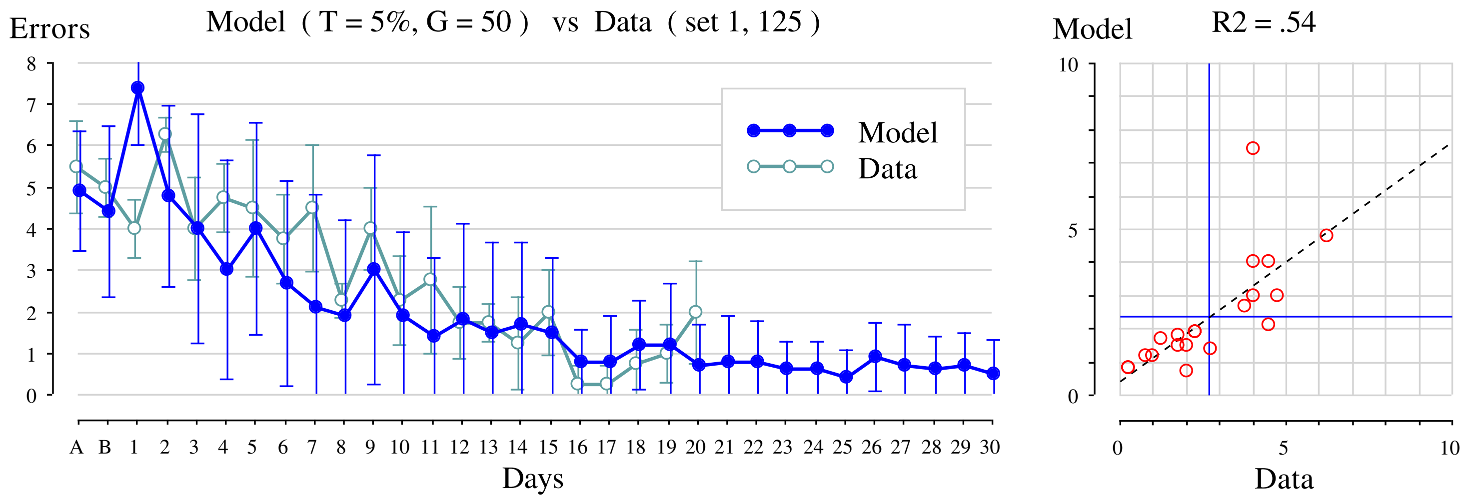
USING ENTROPY FOR CONTROL

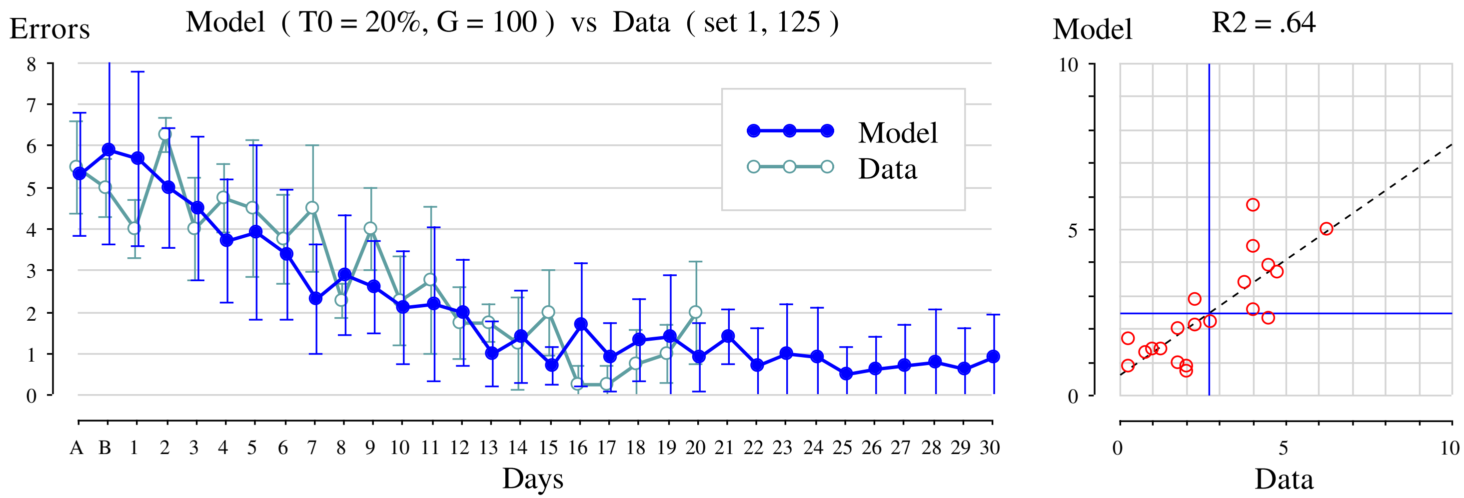
The entropy was fed back to the noise variance:

$$\tau(t) = \tau_0 H_{\mathrm{rel}}(t)$$

- Dynamically changes the randomness making the behaviour more adaptive.
- May explain emotional strategy changes during problem solving (Dörner, 2001).
- Addresses the problem of noise decay towards the end of problem solving (Lebiere, 19??; Taatgen, 2001; Belavkin, Ritter, & Elliman, 1999). Thus, can improve the model fit.

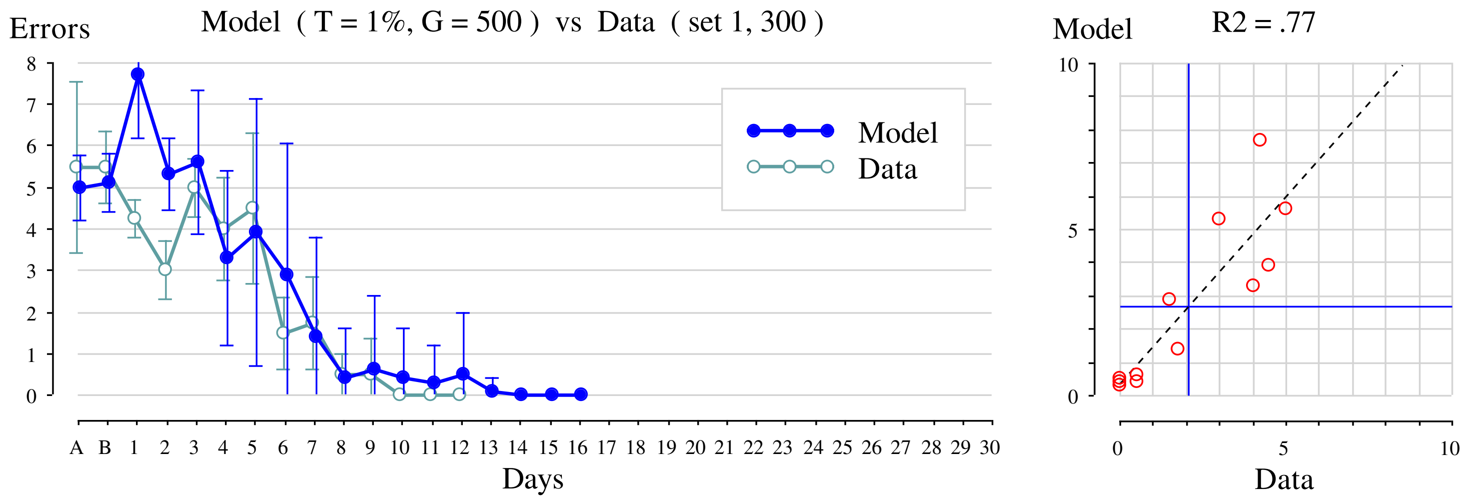
Static noise model

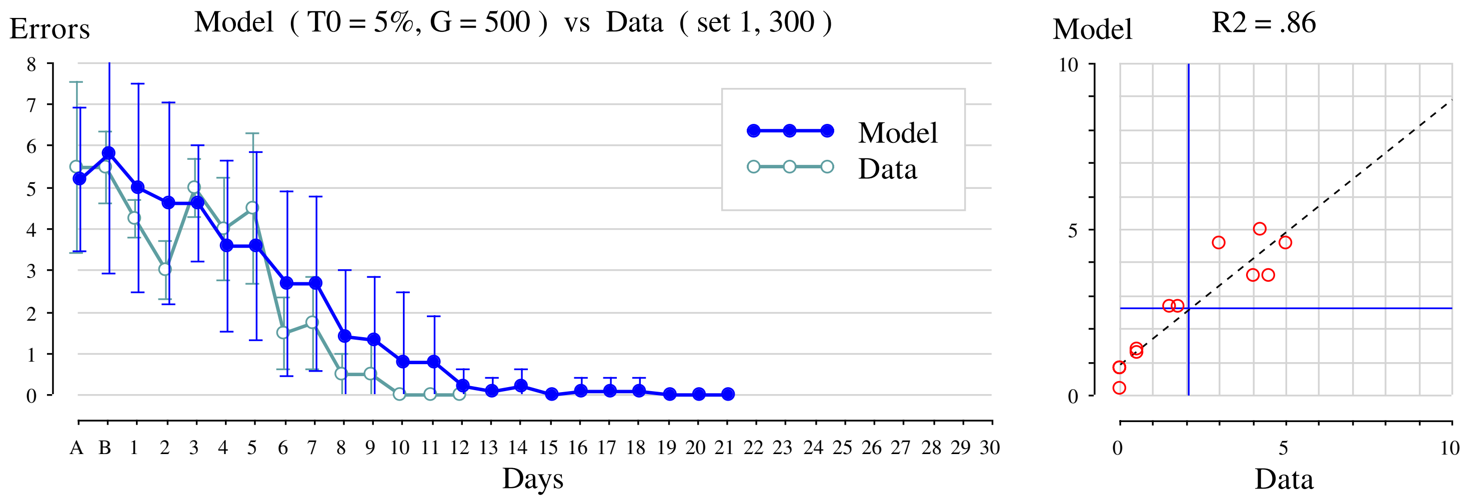




Dynamic noise model

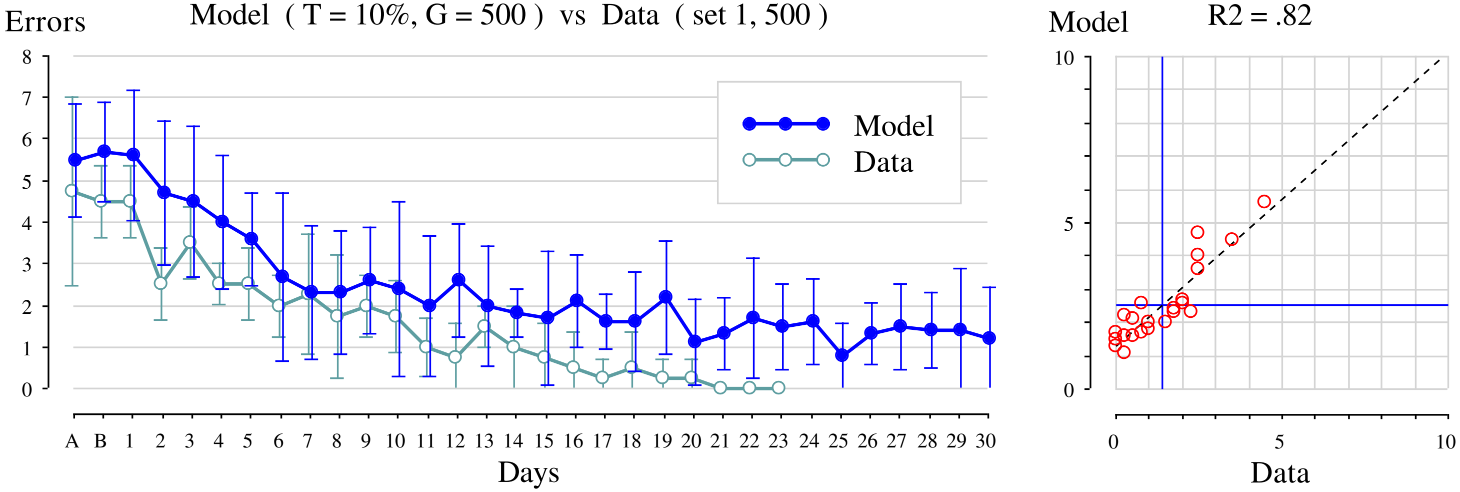
Static noise model

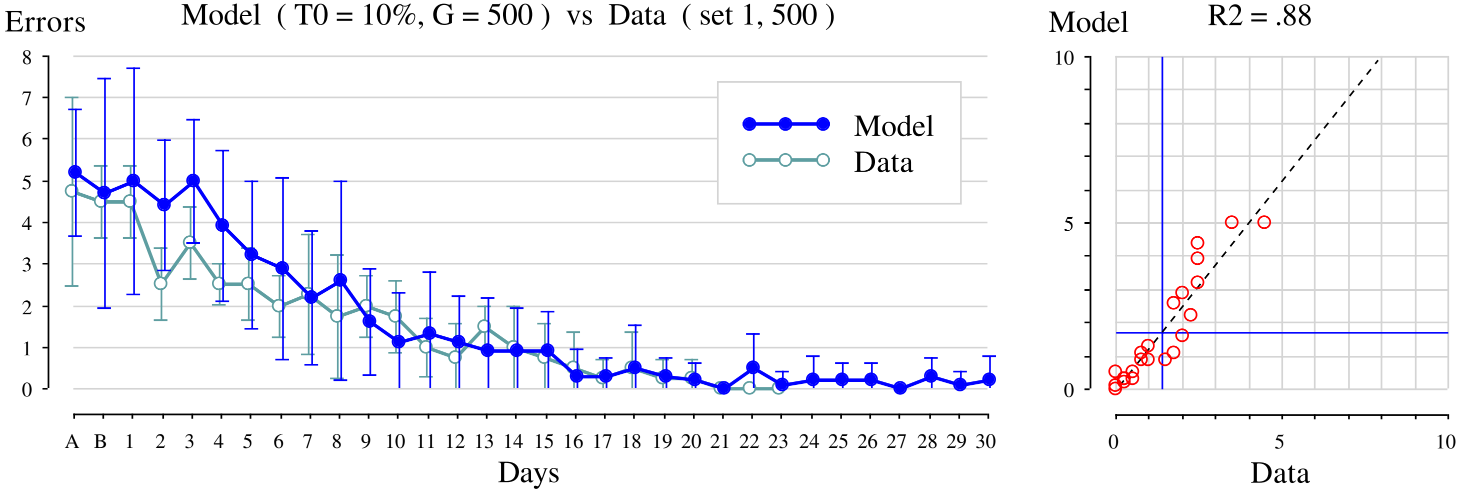




Dynamic noise model

Static noise model





Dynamic noise model

SUMMARY OF MODEL IMPROVEMENT

Comparison of models with static and dynamic noise variance to Yerkes and Dodson data:

	Static noise		Dynamic noise	
Data set	R^2	RMSE	R^2	RMSE
Set I–125	.54	12.2%	.64	10.1%
Set I–300	.77	13.2%	.86	8.8%
Set I–500	.82	12.4%	.88	7.1%

CONCLUSIONS

- It has been shown how to define and calculate the entropy in cognitive architectures (e.g. ACT–R).
- Cognitive architectures provide enough information to estimate the entropy.
- Entropy is a useful for analysing model performance.
- Controlling noise with entropy improved the match of our model to Yerkes and Dodson data.

OPTIMIST: A NEW CONFLICT RESOLUTION

- Instead of utilities uses rates of success estimated from Poisson distribution
- Noise is dynamic and is a function of experience of each rule
- No G (goal value) parameter, but the equivalent estimated costs are dynamic
- Reinforcements (rewards / penalties) are continuous (real–valued) and are determined by the environment, not by the architecture
- Works as an overlay for ACT–R with existing models

http://www.cs.nott.ac.uk/~rvb/

http://www.cs.mdx.ac.uk/~rvb/

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