Permissions and concurrency: a breakthrough and a Grand Challenge

Richard Bornat (Middlesex, UK)

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- (why do the low life do without types? what can we do for them?)

A Rough History

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The world doesn't yet believe in type inference, and the inventor of C++ has never been put on trial.

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$hd([\,])$, anybody?

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Vivid analogy: *"An unfrocked priest is capable of celebrating marriage but forbidden to do so"*
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(BTW, "Ownership types" and/or nulling disposed pointers don't touch this problem.)

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"*We have stipulated that processes should be connected loosely; by this we mean that apart from the (rare) moments of explicit intercommunication, the individual processes themselves are to be regarded as completely independent of each other.*"

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- $A \wedge (B \star \mathrm{true})$ is all A, partly B.

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$$\begin{array}{rll}
\{R^x_E\} & x{:=}E & \{R\} \\
\{x \mapsto _\} & [x]{:=}E & \{x \mapsto E\} \\
\{E' \mapsto E\} & x{:=}[E'] & \{E' \mapsto E \wedge x = E\} \ (x \text{ not free in } E, E') \\
\{\mathbf{emp}\} & x{:=}\mathrm{new}(E) & \{x \mapsto E\} \\
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\end{array}$$

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- Both proved sound by Brookes.
- A version of the CCR rule covers semaphores, in which C is either $m := m + 1$ or $m := m - 1$.

The ownership trick (O'Hearn 2002)

$$\text{resource-bundle } r : \text{Vars } \mathit{full}, b;\ \mathit{full} := \text{false};$$

$$\left(\begin{array}{l||l}
x := \text{new}(); & \text{with } r \text{ when } \mathit{full} \text{ do} \\
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- We want to specify passivity by specifying a read-only resource.
- We require that a program, given a read-only resource, cannot change its contents.

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- Concurrent read permissions must be $(\star)$ separable, because of the concurrency rule.

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- A program which doesn't keep account leaks resource.

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- He associates a number z with each permission: $z = 1$ total; $0 < z < 1$ read-only.
- Fractional permissions are specification-only (cf. types).

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- Boyland (Wisconsin) developed a means of permission accounting in disjoint concurrency, dealing with variables and heap locations.
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- Fractional permissions are specification-only (cf. types).
- In practice the arithmetic is very easy: fractions are simpler to use than (e.g.) sets of binary trees.
- The magnitude of non-integral fractions doesn't matter, except as a matter of accounting.

A fractional model (Calcagno, O'Hearn)

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- Heaps are partial maps from Nat to (int, fraction). (Previously Nat to int.)
- A simpler model – just read / total permissions – fails to account and doesn't have the frame property.

Proof theory

Proof theory

$$
\begin{aligned}
E \xmapsto[z]{} E' \quad &\Rightarrow \quad 0 < z \leq 1 \\
E \xmapsto[z+z']{} E' \wedge z > 0 \wedge z' > 0 \;&\Longleftrightarrow\; E \xmapsto[z]{} E' \star E \xmapsto[z']{} E'
\end{aligned}
$$

Proof theory

$$E \xmapsto[z]{} E' \quad \Rightarrow \quad 0 < z \leq 1$$

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$$\begin{array}{rll}
\{R^x_E\} & x{:=}E & \{R\} \\
\{E' \xmapsto[1]{} _\} & [E']{:=}E & \{E' \xmapsto[1]{} E\} \\
\{E' \xmapsto[z]{} E\} & x{:=}[E'] & \{E' \xmapsto[z]{} E \wedge x = E\} \text{ (x not free in E, E')} \\
\{\mathbf{emp}\} & x{:=}\mathrm{new}(E) & \{x \xmapsto[1]{} E\} \\
\{E \xmapsto[1]{} _\} & \text{dispose}\, E & \{\mathbf{emp}\}
\end{array}$$

Proof theory

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\{E \underset{1}{\mapsto} _\} & \mathrm{dispose}\, E & \{\mathbf{emp}\}
\end{array}$$

- Not (yet) proved sound by Brookes. (But surely ...)

Proof

$\{\mathbf{emp}\}$
$x := \mathrm{new}();$

$[x] := 1;$

$$\left(y := [x] \qquad\qquad \middle\| \; z := [x] + 1 \qquad\qquad \right);$$

$\mathrm{dispose}\, x$
$\{\mathbf{emp} \wedge y = 1 \wedge z = 2\}$

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\end{array}
$$

- That is exactly how hard it is to use fractional permissions.

UnProof

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 & \\
\mathrm{dispose}\, x & z := [x] + 1 \\
 &
\end{array}\right) \\
 \\
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\{??\} &
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Passivity and fractions

Termination Monotonicity: if C must terminate normally in h and $h \star h'$ is defined, then C must terminate normally in $h \star h'$.

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- – i.e. it won't terminate in $10 \xmapsto[1.0]{} N$.
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- Thus we have passivity!

Permission counting

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- Some programs count permissions: e.g. pipeline multicasting, readers-and-writers.
- Permission counting is not specification-only.

Readers and Writers (Courtois et.al. 1972)

$\mathrm{P}(\mathit{read});$

$\quad \text{if } \mathit{count} = 0 \text{ then } \mathrm{P}(\mathit{write})$

$\qquad\qquad \text{else skip fi};$

$\quad \mathit{count}{+} := 1;$

$\mathrm{V}(\mathit{read});$

... reading happens here ...

$\mathrm{P}(\mathit{read});$

$\quad \mathit{count}{-} := 1;$

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Readers and writers (CCR version)

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with *read* when true do
  if $count = 0$ then $\mathrm{P}(write)$
    else skip fi;
  $count{+}{:=}\,1$
od;

... reading happens here ...

with *read* when $count > 0$ do
  $count{-}{:=}\,1$;
  if $count = 0$ then $\mathrm{V}(write)$
    else skip fi
od

$\mathrm{P}(write)$;

... writing happens here ...

$\mathrm{V}(write)$

Readers and writers (CCR version)

$\{\mathbf{emp}\}$

with read when true do

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    else skip fi;

  $\mathit{count}{+}{:=}\ 1$

od;

$\{z \rightarrowtail N\}$

  ... reading happens here ...

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  $\mathit{count}{-}{:=}\ 1;$

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- 0 is total permission.
- $E \overset{i}{\mapsto} E' \star E \overset{j}{\mapsto} E' = \begin{cases} \text{undefined} & i \geq 0 \wedge j \geq 0 \\ \text{undefined} & (i \geq 0 \vee j \geq 0) \wedge i + j < 0 \\ E \overset{i+j}{\longmapsto} E' & \text{otherwise} \end{cases}$

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- $E \rightarrowtail E'$ is a notational convenience for $E \xmapsto{-1} E'$.
- We have passivity (same proof as before).

Proof theory

Proof theory

$$E \xmapsto{n} E' \quad \Rightarrow \quad n \geq 0$$

$$E \xmapsto{n} E' \iff E \xmapsto{n+1} E' \star E \rightarrowtail E'$$

Proof theory

$$E \overset{n}{\longmapsto} E' \;\Rightarrow\; n \geq 0$$

$$E \overset{n}{\longmapsto} E' \iff E \overset{n+1}{\longmapsto} E' \star E \rightarrowtail E'$$

$$
\begin{array}{rll}
\{R^x_E\} & x{:=}E & \{R\} \\
\{E' \overset{0}{\longmapsto} \_\} & [x]{:=}E & \{E' \overset{0}{\longmapsto} E\} \\
\{E' \rightarrowtail E\} & x{:=}[E'] & \{E' \rightarrowtail E \wedge x = E\} \text{ (x not free in E, E')} \\
\{\mathbf{emp}\} & x{:=}\mathrm{new}(E) & \{x \overset{0}{\longmapsto} E\} \\
\{E \overset{0}{\longmapsto} \_\} & \mathrm{dispose}\, E & \{\mathbf{emp}\}
\end{array}
$$

Resource safety proof

$write$: if $write = 0$ then **emp** else $z \stackrel{0}{\longmapsto} N$ fi

$read$: if $count = 0$ then **emp** else $z \xmapsto{count} N$ fi

$\{\mathbf{emp}\}$

with $read$ when true do

if $count = 0$ then $\quad$ P($write$)

else $\quad$ skip

fi;

$count{+}{:=}\,1$

od

$\{z \rightarrowtail N\}$

Resource safety proof

$$write : \text{if } write = 0 \text{ then } \mathbf{emp} \text{ else } z \xmapsto{0} N \text{ fi}$$

$$read : \text{if } count = 0 \text{ then } \mathbf{emp} \text{ else } z \xmapsto{count} N \text{ fi}$$

$\{\mathbf{emp}\}$

with *read* when true do

  $\{\text{if } count = 0 \text{ then } \mathbf{emp} \text{ else } z \xmapsto{count} N \text{ fi} \star \mathbf{emp}\}$

  if $count = 0$ then $\mathrm{P}(write)$

   else skip

  fi;

  $count {+}{:=} 1$

od

$\{z \rightarrowtail N\}$

Resource safety proof

$$write : \text{if } write = 0 \text{ then } \mathbf{emp} \text{ else } z \overset{0}{\longmapsto} N \text{ fi}$$
$$read : \text{if } count = 0 \text{ then } \mathbf{emp} \text{ else } z \overset{count}{\longmapsto} N \text{ fi}$$

$\{\mathbf{emp}\}$

with $read$ when true do

$\quad \{\text{if } count = 0 \text{ then } \mathbf{emp} \text{ else } z \overset{count}{\longmapsto} N \text{ fi} \star \mathbf{emp}\}$

$\quad$ if $count = 0$ then $\quad \{\mathbf{emp}\}$ P$(write)$

$\qquad\qquad\qquad$ else $\quad \{z \overset{count}{\longmapsto} N\}$ $\quad$ skip

$\quad$ fi;

$\quad count{+}{:=}\ 1$

od

$\{z \rightarrowtail N\}$

Resource safety proof

$$\mathit{write} : \text{if } \mathit{write} = 0 \text{ then } \mathbf{emp} \text{ else } z \xmapsto{0} N \text{ fi}$$

$$\mathit{read} : \text{if } \mathit{count} = 0 \text{ then } \mathbf{emp} \text{ else } z \xmapsto{\mathit{count}} N \text{ fi}$$

$\{\mathbf{emp}\}$

with *read* when true do

$\{\text{if } \mathit{count} = 0 \text{ then } \mathbf{emp} \text{ else } z \xmapsto{\mathit{count}} N \text{ fi} \star \mathbf{emp}\}$

if $\mathit{count} = 0$ then $\{\mathbf{emp}\}$ $\mathrm{P}(\mathit{write})$ $\{z \xmapsto{0} N\}$

else $\{z \xmapsto{\mathit{count}} N\}$ skip

fi;

$\mathit{count} {+}{:=} 1$

od

$\{z \rightarrowtail N\}$

Resource safety proof

$write : \text{if } write = 0 \text{ then } \mathbf{emp} \text{ else } z \xmapsto{0} N \text{ fi}$
$read : \text{if } count = 0 \text{ then } \mathbf{emp} \text{ else } z \xmapsto{count} N \text{ fi}$

$\{\mathbf{emp}\}$
with *read* when true do
$\{\text{if } count = 0 \text{ then } \mathbf{emp} \text{ else } z \xmapsto{count} N \text{ fi} \star \mathbf{emp}\}$
if $count = 0$ then $\{\mathbf{emp}\}$ P($write$) $\{z \xmapsto{0} N\}$
else $\{z \xmapsto{count} N\}$ skip $\{z \xmapsto{count} N\}$
fi;

$count {+}{:=}\, 1$

od
$\{z \rightarrowtail N\}$

Resource safety proof

$$\mathit{write} : \text{if } \mathit{write} = 0 \text{ then } \mathbf{emp} \text{ else } z \overset{0}{\longmapsto} N \text{ fi}$$
$$\mathit{read} : \text{if } \mathit{count} = 0 \text{ then } \mathbf{emp} \text{ else } z \xmapsto{\mathit{count}} N \text{ fi}$$

$\{\mathbf{emp}\}$

with *read* when true do

$\{\text{if } \mathit{count} = 0 \text{ then } \mathbf{emp} \text{ else } z \xmapsto{\mathit{count}} N \text{ fi} \star \mathbf{emp}\}$

if $\mathit{count} = 0$ then $\{\mathbf{emp}\}$ $\mathrm{P}(\mathit{write})$ $\{z \overset{0}{\longmapsto} N\}$

else $\{z \xmapsto{\mathit{count}} N\}$ skip $\{z \xmapsto{\mathit{count}} N\}$

fi;

$\{z \xmapsto{\mathit{count}} N\}$

$\mathit{count} {+}{:=} 1$

od

$\{z \rightarrowtail N\}$

Resource safety proof

$$\mathit{write} : \text{if } \mathit{write} = 0 \text{ then } \mathbf{emp} \text{ else } z \xmapsto{0} N \text{ fi}$$

$$\mathit{read} : \text{if } \mathit{count} = 0 \text{ then } \mathbf{emp} \text{ else } z \xmapsto{\mathit{count}} N \text{ fi}$$

$\{\mathbf{emp}\}$

with *read* when true do

$\quad \{\text{if } \mathit{count} = 0 \text{ then } \mathbf{emp} \text{ else } z \xmapsto{\mathit{count}} N \text{ fi} \star \mathbf{emp}\}$

$\quad$ if $\mathit{count} = 0$ then $\quad \{\mathbf{emp}\}\ \mathrm{P}(\mathit{write})\ \{z \xmapsto{0} N\}$

$\qquad\qquad$ else $\quad \{z \xmapsto{\mathit{count}} N\}\ \text{skip}\ \{z \xmapsto{\mathit{count}} N\}$

$\quad$ fi;

$\quad \{z \xmapsto{\mathit{count}} N\}$

$\quad \mathit{count} {+}{:=} 1$

$\quad \{z \xmapsto{\mathit{count}-1} N\}$

od

$\{z \rightarrowtail N\}$

Resource safety proof

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$$\mathit{read} : \text{if } \mathit{count} = 0 \text{ then } \mathbf{emp} \text{ else } z \xmapsto{\mathit{count}} N \text{ fi}$$

$\{\mathbf{emp}\}$

with read when true do

$\quad \{\text{if } \mathit{count} = 0 \text{ then } \mathbf{emp} \text{ else } z \xmapsto{\mathit{count}} N \text{ fi} \star \mathbf{emp}\}$

$\quad \text{if } \mathit{count} = 0 \text{ then} \qquad \{\mathbf{emp}\}\ \mathrm{P}(\mathit{write})\ \{z \overset{0}{\longmapsto} N\}$

$\qquad\qquad\qquad \text{else} \quad \{z \xmapsto{\mathit{count}} N\} \quad \text{skip} \quad \{z \xmapsto{\mathit{count}} N\}$

$\quad \text{fi};$

$\quad \{z \xmapsto{\mathit{count}} N\}$

$\quad \mathit{count} +:= 1$

$\quad \{z \xmapsto{\mathit{count}-1} N\} \therefore \{z \xmapsto{\mathit{count}} N \star z \rightarrowtail N\}$

od

$\{z \rightarrowtail N\}$

Do we need two models?

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$$T ::= \mathsf{Lam}\ v\ T \mid \mathsf{App}\ T\ T \mid \mathsf{Var}\ v$$

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$$\begin{aligned}
\mathrm{AST}\ x\ (\mathsf{Lam}\ v\ \beta)\ z &\mathrel{\hat{=}} \exists b.(x \overset{z}{\mapsto} 0, v, b \star \mathrm{AST}\ b\ \beta\ z \\
\mathrm{AST}\ x\ (\mathsf{App}\ \phi\ \alpha)\ z &\mathrel{\hat{=}} \exists f, a. \begin{pmatrix} x \overset{z}{\mapsto} 1, f, a \star \mathrm{AST}\ f\ \phi\ z\ \star \\ \mathrm{AST}\ a\ \alpha\ z \end{pmatrix} \\
\mathrm{AST}\ x\ (\mathsf{Var}\ v)\ z &\mathrel{\hat{=}} x \overset{z}{\mapsto} 2, v
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\mathrm{AST}\ x\ (\mathsf{Var}\ v)\ z &\hat{=} x \overset{z}{\mapsto} 2, v
\end{aligned}$$

$$\begin{aligned}
(\mathsf{Lam}\ v'\ \beta)[\tau/v] &= \begin{cases} \mathsf{Lam}\ v'\ (\beta[\tau/v]) & v \neq v' \\ \mathsf{Lam}\ v'\ \beta & v' = v \end{cases} \\
(\mathsf{App}\ \phi\ \alpha)[\tau/v] &= \mathsf{App}\ (\phi[\tau/v])\ (\alpha[\tau/v]) \\
(\mathsf{Var}\ v')[\tau/v] &= \begin{cases} \mathsf{Var}\ v' & v \neq v' \\ \tau & v = v' \end{cases}
\end{aligned}$$

Parallel tree rewriting

$\mathit{subst}\ x\ y\ v =$

$\quad \text{if } [x] = 0 \text{ then}$ // Lam

$\qquad \text{if } [x+1] \neq v \text{ then } [x+2] := \mathit{subst}\ [x+2]\ y\ v \text{ else } \mathit{skip} \text{ fi};$

$\qquad x$

$\quad \text{elsf } [x] = 1 \text{ then}$ // App – do it in parallel

$\qquad \big([x+1] := \mathit{subst}\ [x+1]\ y\ v \,\|\, [x+2] := \mathit{subst}\ [x+2]\ y\ v)\big)\ ;$

$\qquad x$

$\quad \text{elsf } [x+1] = v \text{ then}$ // Var, same v

$\qquad \text{dispose } x; \text{dispose}(x+1); \text{new}(2, \mathit{copy}\ y)$

$\quad \text{else}$ // Var, different v

$\qquad x$

$\quad \text{fi}$

Parallel tree rewriting

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    $\big([x+1] := \mathit{subst}\ [x+1]\ y\ v \,\big\|\, [x+2] := \mathit{subst}\ [x+2]\ y\ v)\big)$;

    x

  elsf $[x+1] = v$ then // Var, same v

    dispose x; dispose$(x+1)$; new$(2, \mathit{copy}\ y)$

  else // Var, different v

    x

  fi

– proof easy with fractions, ridiculous with counting permissions; readers and writers swings the other way.

Parallel tree rewriting

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  elsf $[x+1] = v$ then // Var, same v

    dispose x; dispose$(x+1)$; new$(2, \mathit{copy}\ y)$

  else // Var, different v

    x

  fi

– proof easy with fractions, ridiculous with counting permissions; readers and writers swings the other way.

We need more than one model!

Passivity and concurrency

- If I have $x \xmapsto[0.5]{} \_$, I can be sure that you can't write to it.

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- Moral: keep your hand on your ha'penny; don't give them everything you've got.

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- In the concurrent/modular case, you might have the other half, or get it temporarily from elsewhere.
- Moral: keep your hand on your ha'penny; don't give them everything you've got.
- (Same applies to counting permissions.)

We don't understand recursive definitions any more

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$$
\begin{aligned}
\mathrm{tree}\ \mathrm{nil}\ \mathsf{Empty} &\mathrel{\hat{=}} \mathbf{emp} \\
\mathrm{tree}\ t\ (\mathsf{Tip}\ \alpha) &\mathrel{\hat{=}} t \mapsto 0, \alpha \\
\mathrm{tree}\ t\ (\mathsf{Node}\ \lambda\ \rho) &\mathrel{\hat{=}} \exists l, r \cdot \big(t \mapsto 1, l, r\ \star \mathrm{tree}\ l\ \lambda \star \mathrm{tree}\ r\ \rho\big)
\end{aligned}
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\end{aligned}
$$

$$
\begin{aligned}
\mathrm{ztree}\ z\ \mathrm{nil}\ \mathsf{Empty} &\hat{=} \mathbf{emp} \\
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\end{aligned}
$$

$x \underset{0.5}{\longmapsto} 1, l, l \star l \underset{1.0}{\longmapsto} 0, 3$ satisfies $\mathrm{ztree}\ 0.5\ x\ (\mathsf{Node}\ (\mathsf{Tip}\ 3)\ (\mathsf{Tip}\ 3))$
(and we can write to it)!!

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\mathrm{ztree}\ z\ t\ (\mathsf{Node}\ \lambda\ \rho) &\mathrel{\hat{=}} \exists l, r \cdot \Big(t \xmapsto[z]{} 1, l, r\ \star \mathrm{ztree}\ z\ l\ \lambda \star \mathrm{ztree}\ z\ r\ \rho\Big)
\end{aligned}
$$

$x \xmapsto[0.5]{} 1, l, l \star l \xmapsto[1.0]{} 0, 3$ satisfies $\mathrm{ztree}\ 0.5\ x\ (\mathsf{Node}\ (\mathsf{Tip}\ 3)\ (\mathsf{Tip}\ 3))$ (and we can write to it)!!

We have $\mathrm{ztree}\ (z + z')\ t\ \tau \iff \mathrm{ztree}\ z\ t\ \tau \star \mathrm{ztree}\ z'\ t\ \tau$, but sometimes only vacuously.

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\end{aligned}$$

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\end{aligned}$$

$x \underset{0.5}{\longmapsto} 1, l, l \star l \underset{1.0}{\longmapsto} 0, 3$ satisfies $\mathrm{ztree}\ 0.5\ x\ (\mathsf{Node}\ (\mathsf{Tip}\ 3)\ (\mathsf{Tip}\ 3))$ (and we can write to it)!!

We have $\mathrm{ztree}\ (z + z')\ t\ \tau \iff \mathrm{ztree}\ z\ t\ \tau \star \mathrm{ztree}\ z'\ t\ \tau$, but sometimes only vacuously.

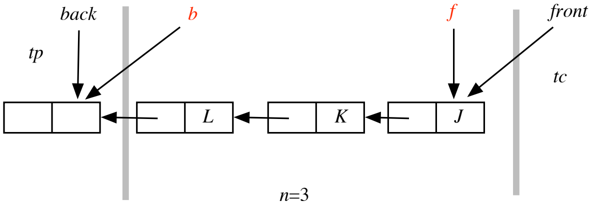
We can write programs which work with $\mathrm{ztree}\ 0.5$, but crash with $\mathrm{ztree}\ 0.499$.

The unbounded buffer

```
begin integer numberOfQueuingPortions,
              bufferManipulation;
      numberOfQueuingPortions := 0;
      bufferManipulation := 1;
      parbegin
      producer:  begin
             again1:  produce next portion;
                      add portion to buffer;
                      V(numberOfQueuingPortions);
                      goto again1
                 end;
      consumer:  begin
             again2:  P(numberOfQueuingPortions);
                      take portion from buffer;
                      process portion taken;
                      goto again2
                 end
      parend
end
```

Proposed and withdrawn in 1965; proved safe, Habermann 1972.

A proof with variables-as-resource



A proof with variables-as-resource

// Producer.

$$\left\{ \begin{array}{l} back, tp, b_{\frac{1}{2}} \vdash \\ \quad back \mapsto \_, \_ \wedge back = b \end{array} \right\}$$

$back.0 := \text{produce}();$
$tp := \text{new}();$
$back.1 := tp;$
$\mathrm{V}(n);$
$back := tp$

$$\left\{ \begin{array}{l} back, tp, b_{\frac{1}{2}} \vdash \\ \quad back \mapsto \_, \_ \wedge back = b \end{array} \right\}$$

$\|$

// Semaphore n

$$\{n, f_{\frac{1}{2}}, b_{\frac{1}{2}} \vdash \text{listseg } n\ f\ b\}$$

$\mathrm{P} : \text{dec}\, n; f := f.2$

$\mathrm{V} : \text{inc}\, n; b := b.2$

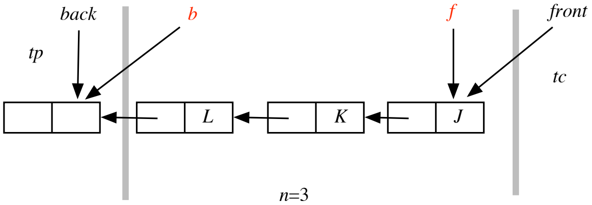
$\|$

// Consumer.

$$\{front, tc, f_{\frac{1}{2}} \vdash front = f\}$$

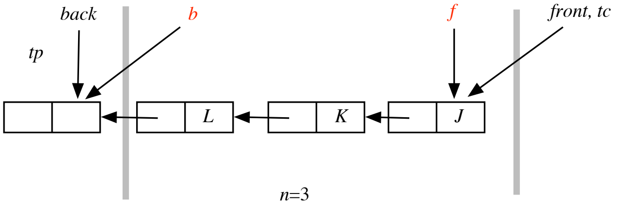
$tc := front;$
$\mathrm{P}(n);$
$front := front.2;$
$\text{consume } tc.0;$
$\text{dispose } tc$

$$\{front, tc, f_{\frac{1}{2}} \vdash front = f\}$$



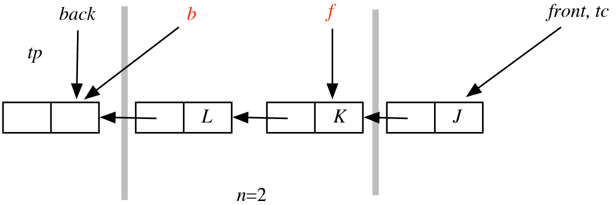
A proof with variables-as-resource

$$
\left(
\begin{array}{l|l|l}
\text{// Producer.} & \text{// Semaphore } n & \text{// Consumer.} \\
\left\{ \begin{array}{l} back, tp, b_{\frac{1}{2}} \vdash \\ \quad back \mapsto \_, \_ \wedge back = b \end{array} \right\} & \{n, f_{\frac{1}{2}}, b_{\frac{1}{2}} \vdash \mathrm{listseg}\ n\ f\ b\} & \{front, tc, f_{\frac{1}{2}} \vdash front = f\} \\
back.0 := \mathrm{produce}(); & & tc := front; \\
tp := \mathrm{new}(); & \mathrm{P} : \mathrm{dec}\, n; f := f.2 & \mathrm{P}(n); \\
back.1 := tp; & & front := front.2; \\
\mathrm{V}(n); & \mathrm{V} : \mathrm{inc}\, n; b := b.2 & \mathrm{consume}\ tc.0; \\
back := tp & & \mathrm{dispose}\ tc \\
\left\{ \begin{array}{l} back, tp, b_{\frac{1}{2}} \vdash \\ \quad back \mapsto \_, \_ \wedge back = b \end{array} \right\} & & \{front, tc, f_{\frac{1}{2}} \vdash front = f\}
\end{array}
\right)
$$



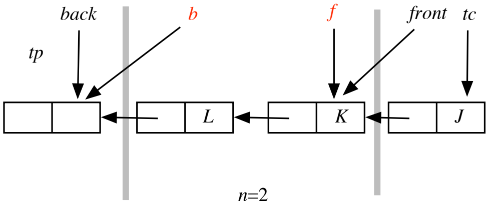
A proof with variables-as-resource

// Producer.	// Semaphore n	// Consumer.
$\left\{\begin{array}{l} back, tp, b_{\frac{1}{2}} \vdash \\ \quad back \mapsto \_, \_ \wedge back = b \end{array}\right\}$	$\{n, f_{\frac{1}{2}}, b_{\frac{1}{2}} \vdash \text{listseg}\ n\ f\ b\}$	$\{front, tc, f_{\frac{1}{2}} \vdash front = f\}$
$back.0 := \text{produce}();$		$tc := front;$
$tp := \text{new}();$	$\text{P} : \text{dec}\, n; f := f.2$	$\text{P}(n);$
$back.1 := tp;$		$front := front.2;$
$\text{V}(n);$	$\text{V} : \text{inc}\, n; b := b.2$	consume $tc.0;$
$back := tp$		dispose tc
$\left\{\begin{array}{l} back, tp, b_{\frac{1}{2}} \vdash \\ \quad back \mapsto \_, \_ \wedge back = b \end{array}\right\}$		$\{front, tc, f_{\frac{1}{2}} \vdash front = f\}$



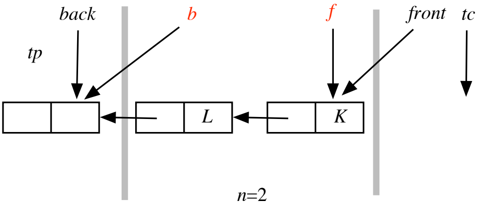
A proof with variables-as-resource

// Producer.	// Semaphore n	// Consumer.
$\left\{ \begin{array}{l} back, tp, b_{\frac{1}{2}} \vdash \\ \quad back \mapsto \_, \_ \wedge back = b \end{array} \right\}$	$\{n, f_{\frac{1}{2}}, b_{\frac{1}{2}} \vdash \text{listseg } n\ f\ b\}$	$\{front, tc, f_{\frac{1}{2}} \vdash front = f\}$
$back.0 := \text{produce}();$		$tc := front;$
$tp := \text{new}();$	$\mathrm{P} : \text{dec}\, n; f := f.2$	$\mathrm{P}(n);$
$back.1 := tp;$		$front := front.2;$
$\mathrm{V}(n);$	$\mathrm{V} : \text{inc}\, n; b := b.2$	$\text{consume } tc.0;$
$back := tp$		$\text{dispose } tc$
$\left\{ \begin{array}{l} back, tp, b_{\frac{1}{2}} \vdash \\ \quad back \mapsto \_, \_ \wedge back = b \end{array} \right\}$		$\{front, tc, f_{\frac{1}{2}} \vdash front = f\}$



A proof with variables-as-resource

$$\left(\begin{array}{l} \text{// Producer.} \\ \left\{ \begin{array}{l} back, tp, b_{\frac{1}{2}} \vdash \\ \quad back \mapsto \_, \_ \wedge back = b \end{array} \right\} \\ back.0 := \text{produce}(); \\ tp := \text{new}(); \\ back.1 := tp; \\ \mathrm{V}(n); \\ back := tp \\ \left\{ \begin{array}{l} back, tp, b_{\frac{1}{2}} \vdash \\ \quad back \mapsto \_, \_ \wedge back = b \end{array} \right\} \end{array} \middle\| \begin{array}{l} \text{// Semaphore } n \\ \{n, f_{\frac{1}{2}}, b_{\frac{1}{2}} \vdash \text{listseg } n\ f\ b\} \\ \\ \mathrm{P} : \text{dec}\, n; f := f.2 \\ \\ \mathrm{V} : \text{inc}\, n; b := b.2 \\ \\ \\ \end{array} \middle\| \begin{array}{l} \text{// Consumer.} \\ \{front, tc, f_{\frac{1}{2}} \vdash front = f\} \\ tc := front; \\ \mathrm{P}(n); \\ front := front.2; \\ \text{consume } tc.0; \\ \text{dispose } tc \\ \{front, tc, f_{\frac{1}{2}} \vdash front = f\} \end{array} \right)$$



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$$
\left(
\begin{array}{l}
\text{// Producer.} \\
\left\{ \begin{array}{l} back, tp, b_{\frac{1}{2}} \vdash \\ \quad back \mapsto \_, \_ \wedge back = b \end{array} \right\} \\
back.0 := \mathrm{produce}(); \\
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\mathrm{V}(n); \\
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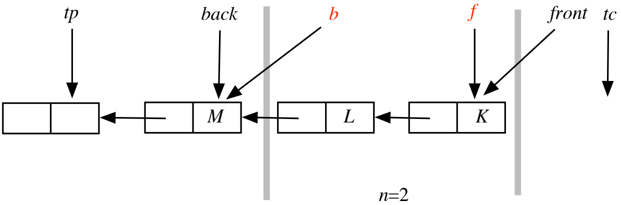
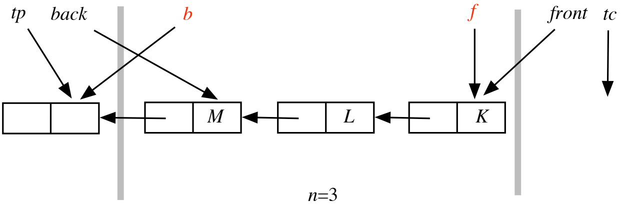
\left\{ \begin{array}{l} back, tp, b_{\frac{1}{2}} \vdash \\ \quad back \mapsto \_, \_ \wedge back = b \end{array} \right\}
\end{array}
\left\|
\begin{array}{l}
\text{// Semaphore } n \\
\{ n, f_{\frac{1}{2}}, b_{\frac{1}{2}} \vdash \mathrm{listseg}\ n\ f\ b \} \\
\\
\mathrm{P} : \mathrm{dec}\, n; f := f.2 \\
\\
\mathrm{V} : \mathrm{inc}\, n; b := b.2 \\
\\
\\
\\
\end{array}
\right\|
\begin{array}{l}
\text{// Consumer.} \\
\{ front, tc, f_{\frac{1}{2}} \vdash front = f \} \\
tc := front; \\
\mathrm{P}(n); \\
front := front.2; \\
\mathrm{consume}\ tc.0; \\
\mathrm{dispose}\ tc \\
\{ front, tc, f_{\frac{1}{2}} \vdash front = f \}
\end{array}
\right)
$$



A proof with variables-as-resource

// Producer.	// Semaphore n	// Consumer.
$\left\{ \begin{array}{l} back, tp, b_{\frac{1}{2}} \vdash \\ \quad back \mapsto \_, \_ \wedge back = b \end{array} \right\}$	$\{n, f_{\frac{1}{2}}, b_{\frac{1}{2}} \vdash \text{listseg } n\ f\ b\}$	$\{front, tc, f_{\frac{1}{2}} \vdash front = f\}$
$back.0 := \text{produce}();$		$tc := front;$
$tp := \text{new}();$	$\text{P} : \text{dec } n; f := f.2$	$\text{P}(n);$
$back.1 := tp;$		$front := front.2;$
$\text{V}(n);$	$\text{V} : \text{inc } n; b := b.2$	$\text{consume } tc.0;$
$back := tp$		$\text{dispose } tc$
$\left\{ \begin{array}{l} back, tp, b_{\frac{1}{2}} \vdash \\ \quad back \mapsto \_, \_ \wedge back = b \end{array} \right\}$		$\{front, tc, f_{\frac{1}{2}} \vdash front = f\}$



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// Producer.

$\left\{ \begin{array}{l} back, tp, b_{\frac{1}{2}} \vdash \\ \quad back \mapsto \_, \_ \land back = b \end{array} \right\}$

$back.0 := \mathrm{produce}();$

$tp := \mathrm{new}();$

$back.1 := tp;$

$\mathrm{V}(n);$

$back := tp$

$\left\{ \begin{array}{l} back, tp, b_{\frac{1}{2}} \vdash \\ \quad back \mapsto \_, \_ \land back = b \end{array} \right\}$

// Semaphore n

$\{n, f_{\frac{1}{2}}, b_{\frac{1}{2}} \vdash \mathrm{listseg}\ n\ f\ b\}$

$\mathrm{P} : \mathrm{dec}\, n; f := f.2$

$\mathrm{V} : \mathrm{inc}\, n; b := b.2$

// Consumer.

$\{front, tc, f_{\frac{1}{2}} \vdash front = f\}$

$tc := front;$

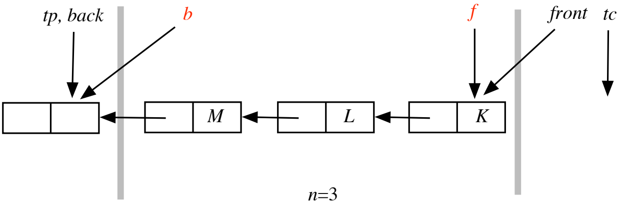
$\mathrm{P}(n);$

$front := front.2;$

$\mathrm{consume}\ tc.0;$

$\mathrm{dispose}\ tc$

$\{front, tc, f_{\frac{1}{2}} \vdash front = f\}$



We can prove it!

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back.0 := \mathrm{produce}(); & & tc := front; \\
tp := \mathrm{new}(); & \mathrm{P} : \mathrm{dec}\, n; f := f.2 & \mathrm{P}(n); \\
back.1 := tp; & & front := front.2; \\
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\end{array}\right)
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Assertion $vs \vdash P$ says "owning variables vs, P holds". P can only mention variables in vs. You can't write to fractionally-owned variables.

We can prove it!

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\left(\begin{array}{l||l||l}
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\end{array}\right)
$$

Assertion $vs \vdash P$ says "owning variables vs, P holds". P can only mention variables in vs. You can't write to fractionally-owned variables.

P can describe separation of the heap: $back \mapsto \_, \_$ describes ownership of a two-word record; $back \mapsto \_, \_ \star front \mapsto \_, \_$ describes ownership of two cons-cells separately.

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If we build it, they will come (as they came for types).