Simpson's 4-slot algorithm, proved in three slides

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(and Matthew Parkinson, ditto)

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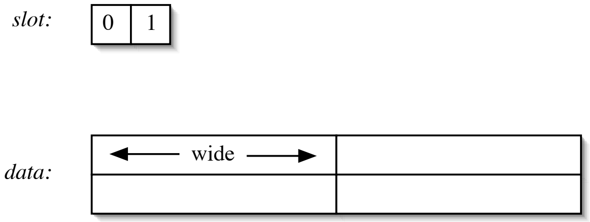
Data structures: a bit array and a wide data array



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```
var    reading, latest : bit
       slot : array bit of bit
       data : array bit of array bit of datatype

procedure  write    (item : datatype);
           var      pair, index : bit;
           begin
                    pair := not(reading);
                    index := not(slot[pair]);
                    data[pair, index] := item;
                    slot[pair] := index;
                    latest := pair
           end;

procedure  read :   datatype;
           var      pair, index : bit;
           begin
                    pair := latest;
                    reading := pair;
                    index := slot[pair];
                    read := data[pair, index]
           end;
```

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- $A \star B$ is separation of heaps; $A \wedge B$, $A \vee B$, $\neg A$, $A \rightarrow B$, $\forall x \cdot P(x)$, $\exists x \cdot P(x)$ are as normal. $A \wedge B$ expresses coincidence of heaps; you don't need to know about $A \mathrel{-\!\!\star} B$.

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- $A \star B$ is separation of heaps; $A \wedge B$, $A \vee B$, $\neg A$, $A \longrightarrow B$, $\forall x \cdot P(x)$, $\exists x \cdot P(x)$ are as normal. $A \wedge B$ expresses coincidence of heaps; you don't need to know about $A \mathrel{-\!\!\star} B$.
- $E \mapsto F_0, F_1$ is just shorthand for $E \mapsto F_0 \star E + 1 \mapsto F_1$.

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- The 'small axioms' of assignment are

$$\{\mathbf{emp}\}\, x := \mathrm{new}()\, \{x \mapsto \_\}$$
$$\{E \mapsto \_\}\ \mathrm{dispose}\, E\, \{\mathbf{emp}\}$$
$$\{R[E/x]\}\, x := E\, \{R\} \quad (\text{the Hoare axiom})$$
$$\{E \mapsto F\}\, x := [E]\, \{x = F \wedge E \mapsto F\} \quad (x \text{ not free in } E, F)$$
$$\{E \mapsto \_\}\, [E] := F\, \{E \mapsto F\}$$

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- The concurrency rule (has horrid side-condition):

$$\frac{\{Q_1\}\, C_1\, \{R_1\} \quad \{Q_2\}\, C_2\, \{R_2\} \quad \ldots \quad \{Q_n\}\, C_n\, \{R_n\}}{\{Q_1 \star Q_2 \star \cdots \star Q_n\}\, C_1 \mid\mid C_2 \mid\mid \cdots \mid\mid C_n\, \{R_1 \star R_2 \star \cdots \star R_n\}}$$

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- The CCR rule (has *atrocious* side condition):

$$\frac{\{(Q \star I_b) \wedge G\}\ C\ \{R \star I_b\}}{\{Q\}\ \text{with}\ b\ \text{when}\ G\ \text{do}\ C\ \text{od}\ \{R\}}$$

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- Permissions (fractions of $\longmapsto$, counts of $\rightarrowtail$) to allow sharing of heap;

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- Permissions (fractions of $\mapsto$, counts of $\rightarrowtail$) to allow sharing of heap;
- Variable permissions, to allow variables to be resource;
- Trivial side conditions;
- No side conditions at all (very new, this!).

Nine lines are now ten, *with added auxiliary proof-variables*

write: with $bundle$ when true do $pair :=$ not$(reading)$; $wuse := pair$ od;
$index :=$ not$(slot[pair])$;
$data[pair, index] := item$;
with $bundle$ when true do $slot[pair] := index$; $wuse := -1$ od;
with $bundle$ when true do $latest := pair$ od

read: with $bundle$ when true do $pair := latest$ od;
with $bundle$ when true do $reading := pair$ od;
with $bundle$ when true do $index := slot[pair]$; $ruse := index$ od;
$read := data[pair, index]$;
with $bundle$ when true do $ruse := -1$ od

What the writer owns

(A point of notation: I've used a special form of $\mapsto$ to describe a heap, just to make the slides easy to read.

For example, $data[pair, index] \mapsto \_$ replaces $data + 2 \star pair + index \mapsto \_$.

There is no change in meaning.)

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There is no change in meaning.)

$$latest_{0.5}, slot_{0.5}, data_{0.33}, wuse_{0.5}, pair, index$$

$$\models \left(\begin{array}{l} slot[0] \xmapsto[0.5]{} \_ \star slot[1] \xmapsto[0.5]{} \_ \star \\ \text{if } wuse \geq 0 \text{ then } data[pair, index] \mapsto \_ \text{ else } \mathbf{emp} \text{ fi} \end{array} \right)$$

What the reader owns

$$
\begin{aligned}
&reading_{0.5}, ruse_{0.5}, data_{0.33}, pair, index \\
&\quad \models \text{if } ruse \geq 0 \text{ then } data[pair, index] \mapsto \_ \text{ else } \mathbf{emp} \text{ fi}
\end{aligned}
$$

The bundle owns the rest

$$latest_{0.5},\ reading_{0.5},\ slot_{0.5},\ data_{0.33},\ wuse_{0.5},\ ruse_{0.5}$$

$$\models \exists s \cdot \left(\begin{array}{l}
slot[0] \xmapsto[0.5]{} s(0) \star slot[1] \xmapsto[0.5]{} s(1) \star \\
\text{if } wuse \geq 0 \wedge ruse \geq 0 \text{ then} \\
\quad data[reading, \text{not}(ruse)] \mapsto _ \star data[wuse, s(wuse)] \mapsto _ \\
\text{elsf } wuse \geq 0 \text{ then} \\
\quad data[wuse, s(wuse)] \mapsto _ \star \\
\quad data[\text{not}(wuse), s(\text{not}(wuse))] \mapsto _ \star data[\text{not}(wuse), \text{not}(s(\text{not}(wuse)))] \mapsto _ \\
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\text{else} \\
\quad data \mapsto _, _, _, _ \\
\text{fi}
\end{array} \right)$$

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with $bundle$ when true do $pair :=$ not$(reading)$; $wuse := pair$ od;

$index :=$ not$(slot[pair])$;

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The reader is even easier than the writer!

$\{\ reading_{0.5}, ruse_{0.5}, data_{0.33}, pair, index \vDash ruse = -1\ \}$

with *bundle* when true do *pair* := *latest* od;

with *bundle* when true do *reading* := *pair* od;

with *bundle* when true do *index* := *slot*[*pair*]; *ruse* := *index* od;

read := *data*[*pair*, *index*];

with *bundle* when true do *ruse* := −1 od

$\{\ reading_{0.5}, ruse_{0.5}, data_{0.33}, pair, index \vDash ruse = -1\ \}$

The reader is even easier than the writer!

$\{\ reading_{0.5}, ruse_{0.5}, data_{0.33}, pair, index \vDash ruse = -1\ \}$
with *bundle* when true do $pair := latest$ od;
$\{\ reading_{0.5}, ruse_{0.5}, data_{0.33}, pair, index \vDash ruse = -1\ \}$
with *bundle* when true do $reading := pair$ od;

with *bundle* when true do $index := slot[pair]; ruse := index$ od;

$read := data[pair, index];$

with *bundle* when true do $ruse := -1$ od
$\{\ reading_{0.5}, ruse_{0.5}, data_{0.33}, pair, index \vDash ruse = -1\ \}$

The reader is even easier than the writer!

$\{\ \mathit{reading}_{0.5}, \mathit{ruse}_{0.5}, \mathit{data}_{0.33}, \mathit{pair}, \mathit{index} \vDash \mathit{ruse} = -1\ \}$

$\quad$ with bundle when true do $\mathit{pair} := \mathit{latest}$ od;

$\{\ \mathit{reading}_{0.5}, \mathit{ruse}_{0.5}, \mathit{data}_{0.33}, \mathit{pair}, \mathit{index} \vDash \mathit{ruse} = -1\ \}$

$\quad$ with bundle when true do $\mathit{reading} := \mathit{pair}$ od;

$\{\ \mathit{reading}_{0.5}, \mathit{ruse}_{0.5}, \mathit{data}_{0.33}, \mathit{pair}, \mathit{index} \vDash \mathit{ruse} = -1 \wedge \mathit{reading} = \mathit{pair}\ \}$

$\quad$ with bundle when true do $\mathit{index} := \mathit{slot}[\mathit{pair}]; \mathit{ruse} := \mathit{index}$ od;

$\quad \mathit{read} := \mathit{data}[\mathit{pair}, \mathit{index}];$

$\quad$ with bundle when true do $\mathit{ruse} := -1$ od

$\{\ \mathit{reading}_{0.5}, \mathit{ruse}_{0.5}, \mathit{data}_{0.33}, \mathit{pair}, \mathit{index} \vDash \mathit{ruse} = -1\ \}$

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with *bundle* when true do $reading := pair$ od;
$\{\ reading_{0.5}, ruse_{0.5}, data_{0.33}, pair, index \vDash ruse = -1 \wedge reading = pair\ \}$
with *bundle* when true do $index := slot[pair]$; $ruse := index$ od;
$\{\ reading_{0.5}, ruse_{0.5}, data_{0.33}, pair, index \vDash ruse \geq 0 \wedge reading = pair \wedge data[pair, index] \mapsto \_\ \}$
$read := data[pair, index]$;

with *bundle* when true do $ruse := -1$ od
$\{\ reading_{0.5}, ruse_{0.5}, data_{0.33}, pair, index \vDash ruse = -1\ \}$

The reader is even easier than the writer!

$\{\ reading_{0.5}, ruse_{0.5}, data_{0.33}, pair, index \vDash ruse = -1\ \}$
with $bundle$ when true do $pair := latest$ od;
$\{\ reading_{0.5}, ruse_{0.5}, data_{0.33}, pair, index \vDash ruse = -1\ \}$
with $bundle$ when true do $reading := pair$ od;
$\{\ reading_{0.5}, ruse_{0.5}, data_{0.33}, pair, index \vDash ruse = -1 \wedge reading = pair\ \}$
with $bundle$ when true do $index := slot[pair]$; $ruse := index$ od;
$\{\ reading_{0.5}, ruse_{0.5}, data_{0.33}, pair, index \vDash ruse \geq 0 \wedge reading = pair \wedge data[pair, index] \mapsto \_\ \}$
$read := data[pair, index]$;

$$\left\{ \begin{array}{l} reading_{0.5}, ruse_{0.5}, data_{0.33}, pair, index \\ \quad \vDash ruse \geq 0 \wedge reading = pair \wedge \exists i \cdot data[pair, index] \mapsto i \wedge read = i \end{array} \right\}$$

with $bundle$ when true do $ruse := -1$ od
$\{\ reading_{0.5}, ruse_{0.5}, data_{0.33}, pair, index \vDash ruse = -1\ \}$

The rest of the reader is too easy to bother with

$$\text{with } \mathit{bundle} \text{ when true do } \mathit{index} := \mathit{slot}[\mathit{pair}]; \mathit{ruse} := \mathit{index}$$

(in the reader) is very very *very* similar to

$$\text{with } \mathit{bundle} \text{ when true do } \mathit{pair} := \text{not}(\mathit{reading}); \mathit{wuse} := \mathit{pair} \text{ od}$$

(which I just showed you in detail from the writer),

so you don’t need to see it.

The rest of the reader is too easy to bother with

with $bundle$ when true do $index := slot[pair]$; $ruse := index$

(in the reader) is very very *very* similar to

with $bundle$ when true do $pair := \text{not}(reading)$; $wuse := pair$ od

(which I just showed you in detail from the writer),

so you don’t need to see it.

And the rest of the reader is trivial.