Dialectical refinement: Rescuing programming from the logicians

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- The novice programming problem

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- The expert programming problem

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Most programming 'languages', *especially* including Java, address the expert programming problem.

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Can trial and error be 'logical'? Or is it illogical guesswork?

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Can we defend program invention? Can we give it a nicer name?

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Can the dialectic reach places that logical refinement cannot? I think so.

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The sensor may write faster than the monitor reads (so there may be missed values).

The monitor may read faster than the sensor writes (so there may be repeated values).

But the monitor *must* always get complete values, not half-written values, not half of one value and half of the next.

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If the sensor does $m := m + 1; s := 0$ then the clock can be seen as fast (by 59 seconds).

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\left(
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\\
\\
\end{array}
\;\middle\|\;
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But atomicity means waiting, and waiting isn't *simple* or even *certain*.

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Data refinement! Now $\lfloor rs \rfloor \preccurlyeq ws \wedge ws_\Omega = c[l]$.

Simplify atomicity in the writer

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\end{array}\right)$$

Stores naturally serialise small reads and writes.

Simplify atomicity in the writer

$$\text{local } c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}$$

$$\left(\begin{array}{l|l} \ldots & \ldots \\ \mathrm{write}(w) \mathrel{\hat{=}} & \mathrm{read}(\,) \mathrel{\hat{=}} \\ \quad \left\langle \begin{array}{l} c[!l] := w; l := !l; \\ ws := ws.w \end{array} \right\rangle & \quad \text{local } y \text{ in} \\ & \qquad \langle y := c[l]; rs := rs.y \rangle; \\ & \qquad \text{return } y \\ & \quad \text{ni} \end{array}\right)$$

$$\text{local } c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}$$

$$\left(\begin{array}{l|l} \ldots & \ldots \\ \mathrm{write}(w) \mathrel{\hat{=}} & \mathrm{read}(\,) \mathrel{\hat{=}} \\ \quad \text{local } wt \text{ in} & \quad \text{local } y \text{ in} \\ \qquad \langle\!\langle wt := !l \rangle\!\rangle; & \qquad \langle y := c[l]; rs := rs.y \rangle; \\ \qquad \langle c[wt] := w \rangle; & \qquad \text{return } y \\ \qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle & \quad \text{ni} \\ \quad \text{ni} & \end{array}\right)$$

Stores naturally serialise small reads and writes. Still $\lfloor rs \rfloor \preccurlyeq ws \wedge ws_\Omega = c[l]$.

(Can't) simplify atomicity in the reader

(Can't) simplify atomicity in the reader

$$
\text{local } c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}
$$

$$
\left(\begin{array}{l|l}
\ldots & \ldots \\
\mathrm{write}(w) \mathrel{\hat{=}} & \mathrm{read}(\,) \mathrel{\hat{=}} \\
\quad \text{local } wt \text{ in} & \quad \text{local } y \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; & \qquad \langle y := c[l]; rs := rs.y \rangle; \\
\qquad \langle c[wt] := w \rangle; & \qquad \text{return } y \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle & \quad \text{ni} \\
\quad \text{ni} &
\end{array}\right)
$$

(Can't) simplify atomicity in the reader

$$\text{local } c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}$$

$$\left(\begin{array}{l|l}
\ldots & \ldots \\
\text{write}(w) \mathrel{\hat{=}} & \text{read}(\,) \mathrel{\hat{=}} \\
\quad \text{local } wt \text{ in} & \quad \text{local } y \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; & \qquad \langle y := c[l]; rs := rs.y \rangle; \\
\qquad \langle c[wt] := w \rangle; & \qquad \text{return } y \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle & \quad \text{ni} \\
\quad \text{ni} &
\end{array}\right)$$

$$\text{local } c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}$$

$$\left(\begin{array}{l|l}
\ldots & \ldots \\
\text{write}(w) \mathrel{\hat{=}} & \text{read}(\,) \mathrel{\hat{=}} \\
\quad \text{local } wt \text{ in} & \quad \text{local } y, rt \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; & \qquad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad \langle c[wt] := w \rangle; & \qquad \langle y := c[rt]; rs := rs.y \rangle; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle & \qquad \text{return } y \\
\quad \text{ni} & \quad \text{ni}
\end{array}\right)$$

(Can't) simplify atomicity in the reader

$$\mathsf{local}\ c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle\ \mathsf{in}$$
$$\left(\begin{array}{l} \ldots \\ \mathrm{write}(w) \mathrel{\hat{=}} \\ \quad \mathsf{local}\ wt\ \mathsf{in} \\ \qquad \langle\!\langle wt := !l \rangle\!\rangle; \\ \qquad \langle c[wt] := w \rangle; \\ \qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle \\ \quad \mathsf{ni} \end{array} \;\middle\|\; \begin{array}{l} \ldots \\ \mathrm{read}(\,) \mathrel{\hat{=}} \\ \quad \mathsf{local}\ y\ \mathsf{in} \\ \qquad \langle y := c[l]; rs := rs.y \rangle; \\ \qquad \mathsf{return}\ y \\ \quad \mathsf{ni} \end{array}\right)$$

$$\mathsf{local}\ c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle\ \mathsf{in}$$
$$\left(\begin{array}{l} \ldots \\ \mathrm{write}(w) \mathrel{\hat{=}} \\ \quad \mathsf{local}\ wt\ \mathsf{in} \\ \qquad \langle\!\langle wt := !l \rangle\!\rangle; \\ \qquad \langle c[wt] := w \rangle; \\ \qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle \\ \quad \mathsf{ni} \end{array} \;\middle\|\; \begin{array}{l} \ldots \\ \mathrm{read}(\,) \mathrel{\hat{=}} \\ \quad \mathsf{local}\ y, rt\ \mathsf{in} \\ \qquad \langle\!\langle rt := l \rangle\!\rangle; \\ \qquad \langle y := c[rt]; rs := rs.y \rangle; \\ \qquad \mathsf{return}\ y \\ \quad \mathsf{ni} \end{array}\right)$$

Looks plausible, but it's broken.

(Can't) simplify atomicity in the reader

$$\text{local } c[2] = (null, null), l = 0, \color{red}{ws} = \langle\,\rangle.null, \color{red}{rs} = \langle\,\rangle \text{ in}$$

$$\left(\begin{array}{l||l}
\ldots & \ldots \\
\text{write}(w) \mathrel{\hat{=}} & \text{read}(\,) \mathrel{\hat{=}} \\
\quad \text{local } wt \text{ in} & \quad \text{local } y \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; & \qquad \langle y := c[l]; \color{red}{rs} := \color{red}{rs}.y \rangle; \\
\qquad \langle c[wt] := w \rangle; & \qquad \text{return } y \\
\qquad \langle\!\langle l := wt; \color{red}{ws} := \color{red}{ws}.w \rangle\!\rangle & \quad \text{ni} \\
\quad \text{ni} &
\end{array}\right)$$

$$\text{local } c[2] = (null, null), l = 0, \color{red}{ws} = \langle\,\rangle.null, \color{red}{rs} = \langle\,\rangle \text{ in}$$

$$\left(\begin{array}{l||l}
\ldots & \ldots \\
\text{write}(w) \mathrel{\hat{=}} & \text{read}(\,) \mathrel{\hat{=}} \\
\quad \text{local } wt \text{ in} & \quad \text{local } y, rt \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; & \qquad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad \langle c[wt] := w \rangle; & \qquad \langle y := c[rt]; \color{red}{rs} := \color{red}{rs}.y \rangle; \\
\qquad \langle\!\langle l := wt; \color{red}{ws} := \color{red}{ws}.w \rangle\!\rangle & \qquad \text{return } y \\
\quad \text{ni} & \quad \text{ni}
\end{array}\right)$$

Looks plausible, but it's broken. Still $\color{red}{ws}_\Omega = c[l]$

(Can't) simplify atomicity in the reader

$$\text{local } c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}$$

$$\left(\begin{array}{l|l}
\begin{array}{l}
\ldots \\
\mathrm{write}(w) \mathrel{\hat{=}} \\
\quad \text{local } wt \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; \\
\qquad \langle c[wt] := w \rangle; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle \\
\quad \text{ni}
\end{array}
&
\begin{array}{l}
\ldots \\
\mathrm{read}(\,) \mathrel{\hat{=}} \\
\quad \text{local } y \text{ in} \\
\qquad \langle y := c[l]; rs := rs.y \rangle; \\
\qquad \text{return } y \\
\quad \text{ni}
\end{array}
\end{array}\right)$$

$$\text{local } c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}$$

$$\left(\begin{array}{l|l}
\begin{array}{l}
\ldots \\
\mathrm{write}(w) \mathrel{\hat{=}} \\
\quad \text{local } wt \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; \\
\qquad \langle c[wt] := w \rangle; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle \\
\quad \text{ni}
\end{array}
&
\begin{array}{l}
\ldots \\
\mathrm{read}(\,) \mathrel{\hat{=}} \\
\quad \text{local } y, rt \text{ in} \\
\qquad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad \langle y := c[rt]; rs := rs.y \rangle; \\
\qquad \text{return } y \\
\quad \text{ni}
\end{array}
\end{array}\right)$$

Looks plausible, but it's broken. Still $ws_\Omega = c[l]$, but no longer $\lfloor rs \rfloor \preccurlyeq ws$.

What goes wrong?

$$\text{local } c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}$$

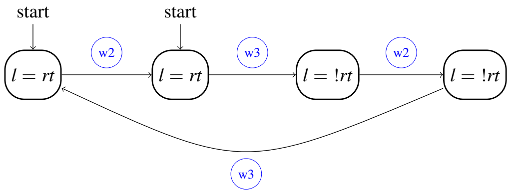
$$\left(\begin{array}{l||l}
\quad \ldots & \quad \ldots \\
\quad \mathrm{write}(w) \mathrel{\hat{=}} & \quad \mathrm{read}(\,) \mathrel{\hat{=}} \\
\qquad \text{local } wt \text{ in} & \qquad \text{local } y, rt \text{ in} \\
\qquad\quad \langle\!\langle wt := !l \rangle\!\rangle; \quad \text{w1} & \qquad\quad \langle\!\langle rt := l \rangle\!\rangle; \quad \text{r1} \\
\qquad\quad \langle c[wt] := w \rangle; \quad \text{w2} & \qquad\quad \langle y := c[rt]; rs := rs.y \rangle; \quad \text{r2} \\
\qquad\quad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle \quad \text{w3} & \qquad\quad \text{return } y \\
\qquad \text{ni} & \qquad \text{ni}
\end{array}\right)$$

What goes wrong?

local $c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle$ in

$$
\left(
\begin{array}{l l || l l}
\ldots & & \ldots & \\
\mathrm{write}(w) \mathrel{\hat{=}} & & \mathrm{read}(\,) \mathrel{\hat{=}} & \\
\quad \textsf{local } wt \textsf{ in} & & \quad \textsf{local } y, rt \textsf{ in} & \\
\qquad \langle\langle wt := !l \rangle\rangle; & \text{w1} & \qquad \langle\langle rt := l \rangle\rangle; & \text{r1} \\
\qquad \langle c[wt] := w \rangle; & \text{w2} & \qquad \langle y := c[rt]; rs := rs.y \rangle; & \text{r2} \\
\qquad \langle\langle l := wt; ws := ws.w \rangle\rangle & \text{w3} & \qquad \textsf{return } y & \\
\quad \textsf{ni} & & \quad \textsf{ni} &
\end{array}
\right)
$$

From the point of view of the reader, after $\langle\langle rt := l \rangle\rangle$, the writer behaves like this finite-state machine:

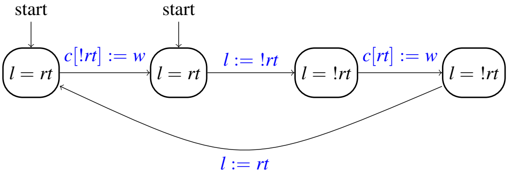


What goes wrong?

local $c[2] = (null, null), l = 0,$ $ws = \langle\,\rangle.null,$ $rs = \langle\,\rangle$ in

$$
\left(
\begin{array}{l|l}
\begin{array}{ll}
\ldots & \\
\text{write}(w) \mathrel{\hat{=}} & \\
\quad \text{local } wt \text{ in} & \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; & \text{w1} \\
\qquad \langle c[wt] := w \rangle; & \text{w2} \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle & \text{w3} \\
\quad \text{ni} &
\end{array}
&
\begin{array}{ll}
\ldots & \\
\text{read}(\,) \mathrel{\hat{=}} & \\
\quad \text{local } y, rt \text{ in} & \\
\qquad \langle\!\langle rt := l \rangle\!\rangle; & \text{r1} \\
\qquad \langle y := c[rt]; rs := rs.y \rangle; & \text{r2} \\
\qquad \text{return } y & \\
\quad \text{ni} &
\end{array}
\end{array}
\right)
$$

From the point of view of the reader, after $\langle\!\langle rt := l \rangle\!\rangle$, the writer behaves like this finite-state machine:



What goes wrong?

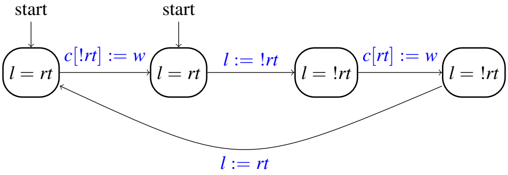
local $c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle$ in

$\ldots$
write$(w)\ \hat{=}$
  local wt in
    $\langle\!\langle wt := !l \rangle\!\rangle;$ w1
    $\langle c[wt] := w \rangle;$ w2
    $\langle\!\langle l := wt;\ ws := ws.w \rangle\!\rangle$ w3
  ni

$\|$

$\ldots$
read$(\,)\ \hat{=}$
  local y, rt in
    $\langle\!\langle rt := l \rangle\!\rangle;$ r1
    $\langle y := c[rt];\ rs := rs.y \rangle;$ r2
    return y
  ni

From the point of view of the reader, after $\langle\!\langle rt := l \rangle\!\rangle$, the writer behaves like this finite-state machine:



If the reader comes in at box 1 or 2 and reads at box 4, it will see the second value written

What goes wrong?

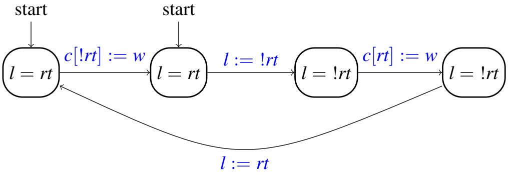
local $c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle$ in

$\ldots$
write$(w) \mathrel{\hat{=}}$
  local wt in
    $\langle\!\langle wt := !l \rangle\!\rangle$; w1
    $\langle c[wt] := w \rangle$; w2
    $\langle\!\langle l := wt; ws := ws.w \rangle\!\rangle$ w3
  ni

$\|$

$\ldots$
read$(\,) \mathrel{\hat{=}}$
  local y, rt in
    $\langle\!\langle rt := l \rangle\!\rangle$; r1
    $\langle y := c[rt]; rs := rs.y \rangle$; r2
    return y
  ni

From the point of view of the reader, after $\langle\!\langle rt := l \rangle\!\rangle$, the writer behaves like this finite-state machine:



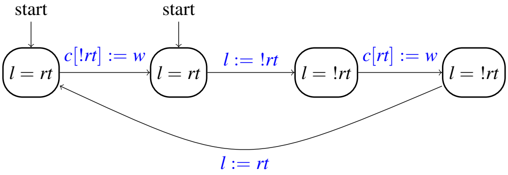
If the reader comes in at box 1 or 2 and reads at box 4, it will see the second value written; if it then comes back quickly, it can see the first thing written!!

What goes wrong?

local $c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle$ in

$$\left(\begin{array}{l|l}
\ldots & \ldots \\
\text{write}(w) \mathrel{\hat{=}} & \text{read}(\,) \mathrel{\hat{=}} \\
\quad \text{local } wt \text{ in} & \quad \text{local } y, rt \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; \quad \text{w1} & \qquad \langle\!\langle rt := l \rangle\!\rangle; \quad \text{r1} \\
\qquad \langle c[wt] := w \rangle; \quad \text{w2} & \qquad \langle y := c[rt]; rs := rs.y \rangle; \quad \text{r2} \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle \quad \text{w3} & \qquad \text{return } y \\
\quad \text{ni} & \quad \text{ni}
\end{array}\right)$$

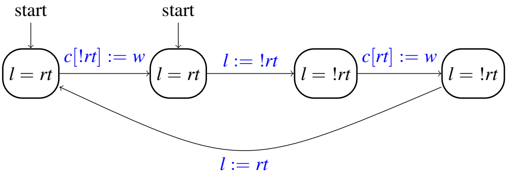
From the point of view of the reader, after $\langle\!\langle rt := l \rangle\!\rangle$, the writer behaves like this finite-state machine:



If the reader comes in at box 1 or 2 and reads at box 4, it will see the second value written; if it then comes back quickly, it can see the first thing written!! Also note $\langle c[rt] := w \rangle \parallel \langle y := c[rt] \rangle$.

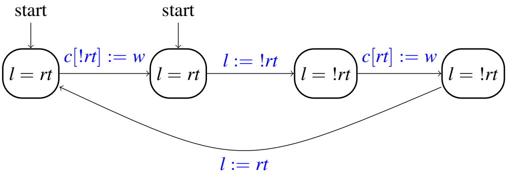
Can we repair it (1)?

All our problems (ordering, collisions) are caused by the third action



Can we repair it (1)?

All our problems (ordering, collisions) are caused by the third action



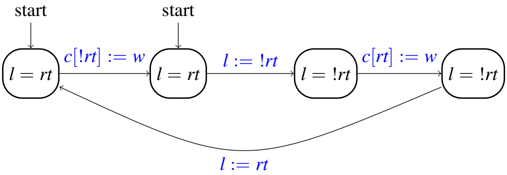
Can we detect when that action happens?

$$
\text{local } c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}
$$

$$
\left(\begin{array}{l|l}
\ldots & \ldots \\
\text{write}(w) \mathrel{\hat{=}} & \text{read}(\,) \mathrel{\hat{=}} \\
\quad \text{local } wt \text{ in} & \quad \text{local } y, rt \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; & \qquad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad \langle c[wt] := w \rangle; & \qquad \langle y := c[rt]; rs := rs.y \rangle; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle & \qquad \text{return } y \\
\quad \text{ni} & \quad \text{ni}
\end{array}\right)
$$

Can we repair it (1)?

All our problems (ordering, collisions) are caused by the third action



Can we detect when that action happens?

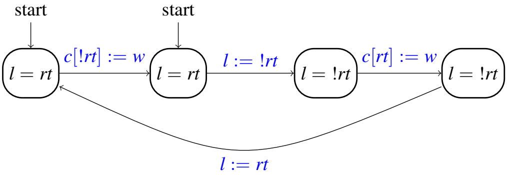
$$\textsf{local}\ c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle\ \textsf{in}$$

$$\left(\begin{array}{l|l}
\ldots & \ldots \\
\mathrm{write}(w) \mathrel{\hat=} & \mathrm{read}(\,) \mathrel{\hat=} \\
\quad \textsf{local}\ wt\ \textsf{in} & \quad \textsf{local}\ y, rt\ \textsf{in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; & \qquad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad \langle c[wt] := w \rangle; & \qquad \langle y := c[rt]; rs := rs.y \rangle; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle & \qquad \textsf{return}\ y \\
\quad \textsf{ni} & \quad \textsf{ni}
\end{array}\right)$$

No, because the writer can't tell the difference between the first and the third actions.

Can we repair it (2)?

All our problems (ordering, collisions) are caused by the third action

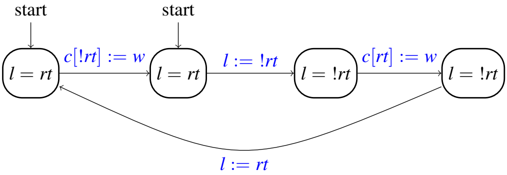


Can we detect when it *might* happen?

$$
\begin{array}{l}
\textsf{local } c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle \textsf{ in} \\
\left(\begin{array}{l|l}
\quad \ldots & \quad \ldots \\
\quad \mathrm{write}(w) \mathrel{\hat=} & \quad \mathrm{read}(\,) \mathrel{\hat=} \\
\quad\quad \textsf{local } wt \textsf{ in} & \quad\quad \textsf{local } y, rt \textsf{ in} \\
\quad\quad\quad \langle\!\langle wt := !l \rangle\!\rangle; & \quad\quad\quad \langle\!\langle rt := l \rangle\!\rangle; \\
\quad\quad\quad \langle c[wt] := w \rangle; & \quad\quad\quad \langle y := c[rt]; rs := rs.y \rangle; \\
\quad\quad\quad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle & \quad\quad\quad \textsf{return } y \\
\quad\quad \textsf{ni} & \quad\quad \textsf{ni}
\end{array}\right)
\end{array}
$$

Can we repair it (2)?

All our problems (ordering, collisions) are caused by the third action



Can we detect when it *might* happen?

local $c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle$ in

$$\left(\begin{array}{l||l}
\quad \ldots & \quad \ldots \\
\quad \mathrm{write}(w) \mathrel{\hat{=}} & \quad \mathrm{read}(\,) \mathrel{\hat{=}} \\
\qquad \textsf{local } wt \textsf{ in} & \qquad \textsf{local } y, rt \textsf{ in} \\
\qquad\quad \langle\!\langle wt := !l \rangle\!\rangle; & \qquad\quad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad\quad \langle c[wt] := w \rangle; & \qquad\quad \langle y := c[rt]; rs := rs.y \rangle; \\
\qquad\quad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle & \qquad\quad \textsf{return } y \\
\qquad \textsf{ni} & \qquad \textsf{ni}
\end{array}\right)$$

Yes: it becomes possible after the second action.

A repair (back to correctness)

The writer signals when disaster becomes possible; the reader incorporates the signal in its answer.

$$\text{local } c[2] = (null, null), l = 0, ok, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}$$

$$\left(\begin{array}{l|l}
\begin{array}{l}
\ldots \\
\text{write}(w) \mathrel{\hat=} \\
\quad \text{local } wt \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; \\
\qquad \langle c[wt] := w \rangle; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle; \\
\qquad \langle\!\langle ok := \text{false} \rangle\!\rangle; \\
\quad \text{ni}
\end{array}
&
\begin{array}{l}
\ldots \\
\text{read}(\,) \mathrel{\hat=} \\
\quad \text{local } y, rt \text{ in} \\
\qquad \langle\!\langle ok := \text{true} \rangle\!\rangle; \\
\qquad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad \langle y := c[rt] \rangle; \\
\qquad \langle\!\langle rt = ok; rs := rs.(rt, y) \rangle\!\rangle; \\
\qquad \text{return } (rt, y) \\
\quad \text{ni}
\end{array}
\end{array}\right)$$

A repair (back to correctness)

The writer signals when disaster becomes possible; the reader incorporates the signal in its answer.

$$\mathsf{local}\ c[2] = (null, null), l = 0, ok, ws = \langle\,\rangle.null, rs = \langle\,\rangle\ \mathsf{in}$$

$$\left(\begin{array}{l|l}
\ldots & \ldots \\
\text{write}(w) \mathrel{\hat{=}} & \text{read}(\,) \mathrel{\hat{=}} \\
\quad \mathsf{local}\ wt\ \mathsf{in} & \quad \mathsf{local}\ y, rt\ \mathsf{in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; & \qquad \langle\!\langle ok := \text{true} \rangle\!\rangle; \\
\qquad \langle c[wt] := w \rangle; & \qquad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle; & \qquad \langle y := c[rt] \rangle; \\
\qquad \langle\!\langle ok := \text{false} \rangle\!\rangle; & \qquad \langle\!\langle rt = ok; rs := rs.(rt, y) \rangle\!\rangle; \\
\quad \mathsf{ni} & \qquad \mathsf{return}\ (rt, y) \\
 & \quad \mathsf{ni}
\end{array}\right)$$

– and then we notice that we don't need atomic buffer accesses any more.

A repair (back to correctness)

The writer signals when disaster becomes possible; the reader incorporates the signal in its answer.

$$\mathsf{local}\ c[2] = (null, null), l = 0, ok, ws = \langle\,\rangle.null, rs = \langle\,\rangle\ \mathsf{in}$$

$$\left(\begin{array}{l}
\ldots \\
\mathrm{write}(w) \mathrel{\hat{=}} \\
\quad \mathsf{local}\ wt\ \mathsf{in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; \\
\qquad c[wt] := w; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle; \\
\qquad \langle\!\langle ok := \mathrm{false} \rangle\!\rangle; \\
\quad \mathsf{ni}
\end{array}
\;\Bigg\|\;
\begin{array}{l}
\ldots \\
\mathrm{read}(\,) \mathrel{\hat{=}} \\
\quad \mathsf{local}\ y, rt\ \mathsf{in} \\
\qquad \langle\!\langle ok := \mathrm{true} \rangle\!\rangle; \\
\qquad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad y := c[rt]; \\
\qquad \langle\!\langle rt = ok; rs := rs.(rt, y) \rangle\!\rangle; \\
\qquad \mathsf{return}\ (rt, y) \\
\quad \mathsf{ni}
\end{array}\right)$$

– and then we notice that we don't need atomic buffer accesses any more.

A repair (back to correctness)

The writer signals when disaster becomes possible; the reader incorporates the signal in its answer.

$$\text{local } c[2] = (null, null), l = 0, ok, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}$$

$$\left(\begin{array}{l||l}
\quad \ldots & \quad \ldots \\
\quad \text{write}(w) \mathrel{\hat{=}} & \quad \text{read}(\,) \mathrel{\hat{=}} \\
\quad\quad \text{local } wt \text{ in} & \quad\quad \text{local } y, rt \text{ in} \\
\quad\quad\quad \langle\!\langle wt := !l \rangle\!\rangle; & \quad\quad\quad \langle\!\langle ok := \text{true} \rangle\!\rangle; \\
\quad\quad\quad c[wt] := w; & \quad\quad\quad \langle\!\langle rt := l \rangle\!\rangle; \\
\quad\quad\quad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle; & \quad\quad\quad y := c[rt]; \\
\quad\quad\quad \langle\!\langle ok := \text{false} \rangle\!\rangle; & \quad\quad\quad \langle\!\langle rt = ok; rs := rs.(rt, y) \rangle\!\rangle; \\
\quad\quad \text{ni} & \quad\quad\quad \text{return } (rt, y) \\
 & \quad\quad \text{ni}
\end{array}\right)$$

– and then we notice that we don’t need atomic buffer accesses any more.

If $\widetilde{rs}$ is rs with the $(\text{false}, \_)$ results taken out and the true labels discarded, then we have $\lfloor \widetilde{rs} \rfloor \preccurlyeq ws \wedge ws_{\Omega} = c[l]$.

Summary so far, and a criticism

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We have gone from an atomic single-slot buffer (not wait-free)

Summary so far, and a criticism

We have gone from an atomic single-slot buffer (not wait-free) to an atomic double-slot buffer (ditto)

Summary so far, and a criticism

We have gone from an atomic single-slot buffer (not wait-free) to an atomic double-slot buffer (ditto) to a faulty not so completely atomic double-slot buffer (still not wait-free)

Summary so far, and a criticism

We have gone from an atomic single-slot buffer (not wait-free) to an atomic double-slot buffer (ditto) to a faulty not so completely atomic double-slot buffer (still not wait-free) to a working wait-free non-atomic double-slot buffer that tells us when it's succeeded.

Summary so far, and a criticism

We have gone from an atomic single-slot buffer (not wait-free) to an atomic double-slot buffer (ditto) to a faulty not so completely atomic double-slot buffer (still not wait-free) to a working wait-free non-atomic double-slot buffer that tells us when it's succeeded.

We certainly haven't proceeded by "a sequence of true understatements" (Lakatos);

Summary so far, and a criticism

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We certainly haven't proceeded by "a sequence of true understatements" (Lakatos); we have made at least one "false overstatement"; perhaps we have made a step of "exception barring".

Summary so far, and a criticism

We have gone from an atomic single-slot buffer (not wait-free) to an atomic double-slot buffer (ditto) to a faulty not so completely atomic double-slot buffer (still not wait-free) to a working wait-free non-atomic double-slot buffer that tells us when it's succeeded.

We certainly haven't proceeded by "a sequence of true understatements" (Lakatos); we have made at least one "false overstatement"; perhaps we have made a step of "exception barring".

I see the step that includes the '*ok*' variable as an example of thesis (program) plus antithesis (counter-example) yielding synthesis (repaired program).

Summary so far, and a criticism

We have gone from an atomic single-slot buffer (not wait-free) to an atomic double-slot buffer (ditto) to a faulty not so completely atomic double-slot buffer (still not wait-free) to a working wait-free non-atomic double-slot buffer that tells us when it's succeeded.

We certainly haven't proceeded by "a sequence of true understatements" (Lakatos); we have made at least one "false overstatement"; perhaps we have made a step of "exception barring".

I see the step that includes the '*ok*' variable as an example of thesis (program) plus antithesis (counter-example) yielding synthesis (repaired program).

But what use is the pair (false, *something*)? What can a user do but ignore *something* and try to read again?

A more honest repair

local $c[2] = (null, null), l = 0, ok, ws = \langle\,\rangle.null, rs = \langle\,\rangle$ in

$$
\left(
\begin{array}{l||l}
\begin{array}{l}
\ldots \\
\mathrm{write}(w) \mathrel{\hat{=}} \\
\quad \textsf{local } wt \textsf{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; \\
\qquad c[wt] := w; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle; \\
\qquad \langle\!\langle ok := \mathrm{false} \rangle\!\rangle; \\
\quad \textsf{ni} \\
\\ \\ \\ \\
\end{array}
&
\begin{array}{l}
\ldots \\
\mathrm{read}(\,) \mathrel{\hat{=}} \\
\quad \textsf{local } y, rt \textsf{ in} \\
\qquad \textsf{do} \\
\qquad\quad \langle\!\langle ok := \mathrm{true} \rangle\!\rangle; \\
\qquad\quad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad\quad y := c[rt]; \\
\qquad\quad \langle\!\langle rt = ok \rangle\!\rangle \\
\qquad \textsf{until } rt; \\
\qquad \langle\!\langle rs := rs.y \rangle\!\rangle; \\
\qquad \textsf{return } y \\
\quad \textsf{ni}
\end{array}
\end{array}
\right)
$$

A more honest repair

$$
\text{local } c[2] = (null, null), l = 0, ok, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}
$$

$$
\left(
\begin{array}{l||l}
\begin{array}{l}
\ldots \\
\mathrm{write}(w) \mathrel{\hat=} \\
\quad \text{local } wt \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; \\
\qquad c[wt] := w; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle; \\
\qquad \langle\!\langle ok := \mathrm{false} \rangle\!\rangle; \\
\quad \text{ni}
\end{array}
&
\begin{array}{l}
\ldots \\
\mathrm{read}(\,) \mathrel{\hat=} \\
\quad \text{local } y, rt \text{ in} \\
\qquad \text{do} \\
\qquad\quad \langle\!\langle ok := \mathrm{true} \rangle\!\rangle; \\
\qquad\quad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad\quad y := c[rt]; \\
\qquad\quad \langle\!\langle rt = ok \rangle\!\rangle \\
\qquad \text{until } rt; \\
\qquad \langle\!\langle rs := rs.y \rangle\!\rangle; \\
\qquad \text{return } y \\
\quad \text{ni}
\end{array}
\end{array}
\right)
$$

Obviously not wait-free.

A more honest repair

local $c[2] = (null, null), l = 0, ok, ws = \langle\,\rangle.null, rs = \langle\,\rangle$ in

$$\left(\begin{array}{l|l}
\begin{array}{l}
\ldots \\
\mathrm{write}(w) \mathrel{\hat{=}} \\
\quad \text{local } wt \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; \\
\qquad c[wt] := w; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle; \\
\qquad \langle\!\langle ok := \mathrm{false} \rangle\!\rangle; \\
\quad \text{ni}
\end{array}
&
\begin{array}{l}
\ldots \\
\mathrm{read}(\,) \mathrel{\hat{=}} \\
\quad \text{local } y, rt \text{ in} \\
\qquad \text{do} \\
\qquad\quad \langle\!\langle ok := \mathrm{true} \rangle\!\rangle; \\
\qquad\quad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad\quad y := c[rt]; \\
\qquad\quad \langle\!\langle rt = ok \rangle\!\rangle \\
\qquad \text{until } rt; \\
\qquad \langle\!\langle rs := rs.y \rangle\!\rangle; \\
\qquad \text{return } y \\
\quad \text{ni}
\end{array}
\end{array}\right)$$

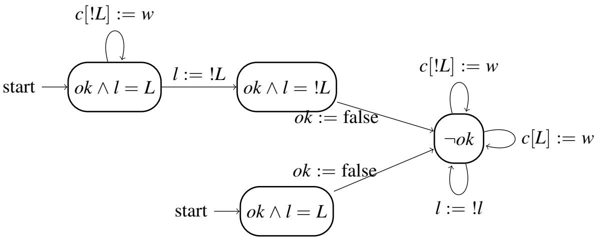
Obviously not wait-free. But otherwise repaired.

A more honest repair

$$
\text{local } c[2] = (null, null), l = 0, ok, {\color{red}ws} = \langle\,\rangle.null, {\color{red}rs} = \langle\,\rangle \text{ in}
$$

$$
\left(
\begin{array}{l||l}
\ldots & \ldots \\
\mathrm{write}(w) \mathrel{\hat{=}} & \mathrm{read}(\,) \mathrel{\hat{=}} \\
\quad \text{local } wt \text{ in} & \quad \text{local } y, rt \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; & \qquad \text{do} \\
\qquad c[wt] := w; & \qquad\quad \langle\!\langle ok := \mathrm{true} \rangle\!\rangle; \\
\qquad \langle\!\langle l := wt; {\color{red}ws} := {\color{red}ws}.w \rangle\!\rangle; & \qquad\quad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad \langle\!\langle ok := \mathrm{false} \rangle\!\rangle; & \qquad\quad y := c[rt]; \\
\quad \text{ni} & \qquad\quad \langle\!\langle rt = ok \rangle\!\rangle \\
 & \qquad \text{until } rt; \\
 & \qquad \langle\!\langle {\color{red}rs} := {\color{red}rs}.y \rangle\!\rangle; \\
 & \qquad \text{return } y \\
 & \quad \text{ni}
\end{array}
\right)
$$

Obviously not wait-free. But otherwise repaired. Finite-state machine now



Try three slots

$$
\text{local } c[2] = (null, null), d = null, l = 0, ok, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}
$$

$$
\left(
\begin{array}{l||l}
\begin{array}{l}
\ldots \\
\mathrm{write}(w) \mathrel{\hat{=}} \\
\quad \text{local } wt \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; \\
\qquad c[wt] := w; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle; \\
\qquad \langle d := w \rangle; \\
\qquad \langle\!\langle ok := \mathrm{false} \rangle\!\rangle; \\
\quad \text{ni}
\end{array}
&
\begin{array}{l}
\ldots \\
\mathrm{read}(\,) \mathrel{\hat{=}} \\
\quad \text{local } y, rt \text{ in} \\
\qquad \langle\!\langle ok := \mathrm{true} \rangle\!\rangle; \\
\qquad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad y := c[rt]; \\
\qquad \langle\!\langle rt = ok \rangle\!\rangle; \\
\qquad \text{if } \neg rt \text{ then } \langle y := d \rangle \text{ else skip fi}; \\
\qquad \langle\!\langle rs := rs.y \rangle\!\rangle; \\
\qquad \text{return } y \\
\quad \text{ni}
\end{array}
\end{array}
\right)
$$

The writer can first write in a side-channel, then signal that mayhem approaches. If it gets the signal, the reader uses the side-channel.

Try three slots

$$\text{local } c[2] = (null, null), d = null, l = 0, ok, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}$$

$$\left(\begin{array}{l|l}
\ldots & \ldots \\
\text{write}(w) \mathrel{\hat{=}} & \text{read}(\,) \mathrel{\hat{=}} \\
\quad \text{local } wt \text{ in} & \quad \text{local } y, rt \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; & \qquad \langle\!\langle ok := \text{true} \rangle\!\rangle; \\
\qquad c[wt] := w; & \qquad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle; & \qquad y := c[rt]; \\
\qquad \langle d := w \rangle; & \qquad \langle\!\langle rt = ok \rangle\!\rangle; \\
\qquad \langle\!\langle ok := \text{false} \rangle\!\rangle; & \qquad \text{if } \neg rt \text{ then } \langle y := d \rangle \text{ else skip fi}; \\
\quad \text{ni} & \qquad \langle\!\langle rs := rs.y \rangle\!\rangle; \\
 & \qquad \text{return } y \\
 & \quad \text{ni}
\end{array}\right)$$

The writer can first write in a side-channel, then signal that mayhem approaches. If it gets the signal, the reader uses the side-channel.

Perhaps this will work ...

Try three slots

$$\text{local } c[2] = (null, null), d = null, l = 0, ok, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}$$

$$\left(\begin{array}{l|l}
\begin{array}{l}
\ldots \\
\mathrm{write}(w) \mathrel{\hat{=}} \\
\quad \text{local } wt \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; \\
\qquad c[wt] := w; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle; \\
\qquad \langle\!\langle wt := ok \rangle\!\rangle; \\
\qquad \text{if } wt \text{ then } \langle d := w \rangle; \langle\!\langle ok := \mathrm{false} \rangle\!\rangle \\
\qquad\quad \text{else skip fi} \\
\quad \text{ni}
\end{array}
&
\begin{array}{l}
\ldots \\
\mathrm{read}(\,) \mathrel{\hat{=}} \\
\quad \text{local } y, rt \text{ in} \\
\qquad \langle\!\langle ok := \mathrm{true} \rangle\!\rangle; \\
\qquad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad y := c[rt]; \\
\qquad \langle\!\langle rt = ok \rangle\!\rangle; \\
\qquad \text{if } \neg rt \text{ then } \langle y := d \rangle \text{ else skip fi}; \\
\qquad \langle\!\langle rs := rs.y \rangle\!\rangle; \\
\qquad \text{return } y \\
\quad \text{ni}
\end{array}
\end{array} \right)$$

The writer can first write in a side-channel, then signal that mayhem approaches. If it gets the signal, the reader uses the side-channel.

Perhaps this will work ... but it's more likely to work if the writer only writes when the reader is asking for it.

Try three slots

$$
\textsf{local}\ c[2] = (null, null), d = null, l = 0, ok, ws = \langle\,\rangle.null, rs = \langle\,\rangle\ \textsf{in}
$$

$$
\left(\begin{array}{l|l}
\begin{array}{l}
\ldots \\
\mathrm{write}(w) \mathrel{\hat{=}} \\
\quad \textsf{local}\ wt\ \textsf{in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; \\
\qquad c[wt] := w; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle; \\
\qquad \langle\!\langle wt := ok \rangle\!\rangle; \\
\qquad \textsf{if}\ wt\ \textsf{then}\ d := w; \langle\!\langle ok := \mathrm{false} \rangle\!\rangle \\
\qquad\quad \textsf{else}\ \mathrm{skip}\ \textsf{fi} \\
\quad \textsf{ni}
\end{array}
&
\begin{array}{l}
\ldots \\
\mathrm{read}(\,) \mathrel{\hat{=}} \\
\quad \textsf{local}\ y, rt\ \textsf{in} \\
\qquad \langle\!\langle ok := \mathrm{true} \rangle\!\rangle; \\
\qquad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad y := c[rt]; \\
\qquad \langle\!\langle rt = ok \rangle\!\rangle; \\
\qquad \textsf{if}\ \neg rt\ \textsf{then}\ y := d\ \textsf{else}\ \mathrm{skip}\ \textsf{fi}; \\
\qquad \langle\!\langle rs := rs.y \rangle\!\rangle; \\
\qquad \textsf{return}\ y \\
\quad \textsf{ni}
\end{array}
\end{array}\right)
$$

The writer can first write in a side-channel, then signal that mayhem approaches. If it gets the signal, the reader uses the side-channel.

Perhaps this will work ... but it's more likely to work if the writer only writes when the reader is asking for it.

And then I notice that they alternate, and I can use *non*-atomic read and write.

Try three slots

$$
\text{local } c[2] = (null, null), d = null, l = 0, ok, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}
$$

$$
\left(
\begin{array}{l||l}
\ldots & \ldots \\
\mathrm{write}(w) \mathrel{\hat=} & \mathrm{read}(\,) \mathrel{\hat=} \\
\quad \text{local } wt \text{ in} & \quad \text{local } y, rt \text{ in} \\
\qquad \langle\!\langle wt := \,!l \rangle\!\rangle; & \qquad \langle\!\langle ok := \mathrm{true} \rangle\!\rangle; \\
\qquad c[wt] := w; & \qquad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle; & \qquad y := c[rt]; \\
\qquad \langle\!\langle wt := ok \rangle\!\rangle; & \qquad \langle\!\langle rt = ok \rangle\!\rangle; \\
\qquad \text{if } wt \text{ then } d := w; \langle\!\langle ok := \mathrm{false} \rangle\!\rangle & \qquad \text{if } \neg rt \text{ then } y := d \text{ else } \mathrm{skip} \text{ fi}; \\
\qquad\quad \text{else } \mathrm{skip} \text{ fi} & \qquad \langle\!\langle rs := rs.y \rangle\!\rangle; \\
\quad \text{ni} & \qquad \text{return } y \\
 & \quad \text{ni}
\end{array}
\right)
$$

The writer can first write in a side-channel, then signal that mayhem approaches. If it gets the signal, the reader uses the side-channel.

Perhaps this will work ... but it's more likely to work if the writer only writes when the reader is asking for it.

And then I notice that they alternate, and I can use *non*-atomic read and write. This is Harris's algorithm, rationally developed.

Recapitulation

$$
\begin{array}{l}
\textsf{local } b = \mathit{null} \textsf{ in} \\
\left(\begin{array}{l|l}
\quad \ldots & \quad \ldots \\
\quad \mathrm{write}(w) \mathrel{\hat{=}} & \quad \mathrm{read}(\,) \mathrel{\hat{=}} \\
\qquad \langle b := w \rangle & \qquad \textsf{local } y \textsf{ in} \\
& \qquad\quad \langle y := b \rangle; \\
& \qquad\quad \textsf{return } y \\
& \qquad \textsf{ni}
\end{array}\right)
\end{array}
$$

– Dijkstra's single-place buffer.

Recapitulation

$$
\begin{array}{l}
\textsf{local } b = null, ws = \langle\,\rangle.null, rs = \langle\,\rangle \textsf{ in} \\
\left(\begin{array}{l|l}
\quad \ldots & \ldots \\
\quad \mathrm{write}(w) \mathrel{\hat{=}} & \mathrm{read}(\,) \mathrel{\hat{=}} \\
\qquad \langle b := w; ws := ws.w \rangle & \quad \textsf{local } y \textsf{ in} \\
 & \qquad \langle y := b; rs := rs.y \rangle; \\
 & \qquad \textsf{return } y \\
 & \quad \textsf{ni}
\end{array}\right)
\end{array}
$$

– with auxiliary ws and rs to show $\lfloor rs \rfloor \preccurlyeq ws \wedge ws_\Omega = b$.

Recapitulation

$$\text{local } c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}$$

$$\left(\begin{array}{l} \ldots \\ \mathrm{write}(w) \mathrel{\hat{=}} \\ \quad \left\langle \begin{array}{l} c[!l] := w; l := !l; \\ ws := ws.w \end{array} \right\rangle \\ \\ \end{array} \;\middle\|\; \begin{array}{l} \ldots \\ \mathrm{read}(\,) \mathrel{\hat{=}} \\ \quad \text{local } y \text{ in} \\ \quad\quad \langle y := c[l]; rs := rs.y \rangle; \\ \quad\quad \text{return } y \\ \quad \text{ni} \end{array} \right)$$

– data refinement to two slots and $\lfloor rs \rfloor \preccurlyeq ws \wedge ws_\Omega = c[l]$.

Recapitulation

$$
\begin{array}{l}
\textsf{local } c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle \textsf{ in} \\
\left(
\begin{array}{l||l}
\begin{array}{l}
\ldots \\
\mathrm{write}(w) \mathrel{\hat{=}} \\
\quad \textsf{local } wt \textsf{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; \\
\qquad \langle c[wt] := w \rangle; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle \\
\quad \textsf{ni}
\end{array}
&
\begin{array}{l}
\ldots \\
\mathrm{read}(\,) \mathrel{\hat{=}} \\
\quad \textsf{local } y \textsf{ in} \\
\qquad \langle y := c[l]; rs := rs.y \rangle; \\
\qquad \textsf{return } y \\
\quad \textsf{ni} \\
\end{array}
\end{array}
\right)
\end{array}
$$

– atomicity refinement in the writer; still $\lfloor rs \rfloor \preccurlyeq ws \wedge ws_\Omega = c[l]$.

Recapitulation

$$
\text{local } c[2] = (null, null), l = 0, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}
$$

$$
\left(\begin{array}{l|l}
\begin{array}{l}
\ldots \\
\text{write}(w) \mathrel{\hat{=}} \\
\quad \text{local } wt \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; \\
\qquad \langle c[wt] := w \rangle; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle \\
\quad \text{ni}
\end{array}
&
\begin{array}{l}
\ldots \\
\text{read}(\,) \mathrel{\hat{=}} \\
\quad \text{local } y, rt \text{ in} \\
\qquad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad \langle y := c[rt]; rs := rs.y \rangle; \\
\qquad \text{return } y \\
\quad \text{ni}
\end{array}
\end{array}\right)
$$

– atomicity refinement in the reader; now $\lfloor rs \rfloor \not\preccurlyeq ws \wedge ws_\Omega = c[l]$.

Recapitulation

$$
\text{local } c[2] = (null, null), l = 0, ok, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}
$$

$$
\left(\begin{array}{l|l}
\begin{array}{l}
\ldots \\
\mathrm{write}(w) \mathrel{\hat{=}} \\
\quad \text{local } wt \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; \\
\qquad c[wt] := w; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle; \\
\qquad \langle\!\langle ok := \mathrm{false} \rangle\!\rangle; \\
\quad \text{ni}
\end{array}
&
\begin{array}{l}
\ldots \\
\mathrm{read}(\,) \mathrel{\hat{=}} \\
\quad \text{local } y, rt \text{ in} \\
\qquad \langle\!\langle ok := \mathrm{true} \rangle\!\rangle; \\
\qquad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad y := c[rt]; \\
\qquad \langle\!\langle rt = ok; rs := rs.(rt, y) \rangle\!\rangle; \\
\qquad \text{return } (rt, y) \\
\quad \text{ni}
\end{array}
\end{array}\right)
$$

– exception barring; now $\lfloor \widetilde{rs} \rfloor \preccurlyeq ws \wedge ws_\Omega = c[l]$.

Recapitulation

$$
\text{local } c[2] = (null, null), l = 0, ok, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}
$$

$$
\left(\begin{array}{l||l}
\begin{array}{l}
\ldots \\
\mathrm{write}(w) \mathrel{\hat{=}} \\
\quad \text{local } wt \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; \\
\qquad c[wt] := w; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle; \\
\qquad \langle\!\langle ok := \mathrm{false} \rangle\!\rangle; \\
\quad \text{ni}
\end{array}
&
\begin{array}{l}
\ldots \\
\mathrm{read}(\,) \mathrel{\hat{=}} \\
\quad \text{local } y, rt \text{ in} \\
\qquad \text{do} \\
\qquad\quad \langle\!\langle ok := \mathrm{true} \rangle\!\rangle; \\
\qquad\quad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad\quad y := c[rt]; \\
\qquad\quad \langle\!\langle rt = ok \rangle\!\rangle \\
\qquad \text{until } rt; \\
\qquad \langle\!\langle rs := rs.y \rangle\!\rangle; \\
\qquad \text{return } y \\
\quad \text{ni}
\end{array}
\end{array}\right)
$$

– more honest exception barring; once again $\lfloor rs \rfloor \preccurlyeq ws \wedge ws_\Omega = c[l]$, but no longer wait-free.

Recapitulation

$$\text{local } c[2] = (null, null), d = null, l = 0, ok, ws = \langle\,\rangle.null, rs = \langle\,\rangle \text{ in}$$

$$\left(\begin{array}{l||l}
\ldots & \ldots \\
\mathrm{write}(w) \mathrel{\hat{=}} & \mathrm{read}(\,) \mathrel{\hat{=}} \\
\quad \text{local } wt \text{ in} & \quad \text{local } y, rt \text{ in} \\
\qquad \langle\!\langle wt := !l \rangle\!\rangle; & \qquad \langle\!\langle ok := \mathrm{true} \rangle\!\rangle; \\
\qquad c[wt] := w; & \qquad \langle\!\langle rt := l \rangle\!\rangle; \\
\qquad \langle\!\langle l := wt; ws := ws.w \rangle\!\rangle; & \qquad y := c[rt]; \\
\qquad \langle\!\langle wt := ok \rangle\!\rangle; & \qquad \langle\!\langle rt = ok \rangle\!\rangle; \\
\qquad \text{if } wt \text{ then } d := w; \langle\!\langle ok := \mathrm{false} \rangle\!\rangle & \qquad \text{if } \neg rt \text{ then } y := d \text{ else } \mathrm{skip} \text{ fi}; \\
\qquad\quad \text{else } \mathrm{skip} \text{ fi} & \qquad \langle\!\langle rs := rs.y \rangle\!\rangle; \\
\quad \text{ni} & \qquad \text{return } y \\
 & \quad \text{ni}
\end{array}\right)$$

– three slots; $\lfloor rs \rfloor \preccurlyeq ws \wedge ws_{\Omega} = c[l]$; wait-free.

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Proof available on application.

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– the reader tells the writer when there's no need to use the side channel.

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In fact the writer writes when *ok* <u>or</u> the reader is asleep. Easy to fix with another auxiliary variable, proof available on request.

Pipelines, caches, weak memory models

In my argument I relied on sequencing of actions and on serialisation of (small) memory accesses.

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Maybe an EPSRC project . . .

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Lakatos's dialectic lives!