Permission accounting in separation logic

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Move over typing: here comes resourcing!

Resourcing is the next formal step towards program safety, following the success of typing. Resourcing is about the amount of resource used by a program; typing is about the kind of resource.

“A well-typed program won’t go wrong” (Milner).

“Well-resourced programs mind their own business” (O’Hearn).
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- Resourcing is about the amount of resource used by a program; typing is about the kind of resource.
- “A well-typed program won’t go wrong” (Milner).
- “Well-resourced programs mind their own business” (O’Hearn).
A program in need of resourcing

\[ P(\text{read}) ; \]
\[ \text{if } \text{count} = 0 \text{ then } P(\text{write}) \]
\[ \text{else skip fi} ; \]
\[ \text{count} + := 1 ; \]
\[ V(\text{read}) ; \]
\[ \ldots \text{ reading happens here } \ldots \]
\[ V(\text{read}) \]
\[ P(\text{read}) ; \]
\[ \text{count} - := 1 ; \]
\[ \text{if } \text{count} = 0 \text{ then } V(\text{write}) \]
\[ \text{else skip fi} ; \]
\[ V(\text{read}) \]

\[ P(\text{write}) ; \]
\[ \ldots \text{ writing happens here } \ldots \]
\[ V(\text{write}) \]
Separation logic

• Just a bastard child of BI (Pym, O’Hearn).
• $E \mapsto \rightarrow E'$ (points to) is permission to read/write/dispose cell at heap address $E$ with contents $E'$.
• Previously $\mapsto \rightarrow$ was ownership; before that a heap predicate (and it still is).
• $\text{emp}$ is no permission.
• $A \star B$ (star) is separation of resource.
• $A \land B$ (and) is identity of resource.
• $A \land (B \star \text{true})$ is all $A$, partly $B$. 
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- $A \land (B \star \text{true})$ is all $A$, partly $B$. 
Framing, hence small axioms
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\[
\begin{align*}
\{Q\} C \{R\} \\
\{P \star Q\} C \{P \star R\}
\end{align*}
\]

(modifies \( C \cap \text{vars } P = \emptyset \))
Framing, hence small axioms

\[ \{ Q \} C \{ R \} \]

\[ \{ P \ast Q \} C \{ P \ast R \} \]

(modifies \( C \cap \text{vars } P = \emptyset \))

\[
\begin{align*}
\{ R^x_E \} & \quad x:=E \\
\{ x \mapsto \_ \} & \quad [x]:=E \\
\{ E' \mapsto E \} & \quad x:=[E'] \\
\{ \text{emp} \} & \quad x:=\text{new}(E) \\
\{ E \mapsto \_ \} & \quad \text{dispose } E
\end{align*}
\]

\[ E' \mapsto E \land x = E \] (\( x \) not free in \( E, E' \))
Concurrent rules
Concurrence rules

\[
\begin{array}{c}
\{Q_1\} C_1 \{R_1\} \cdots \{Q_n\} C_n \{R_n\} \\
\{Q_1 \cdots \cdots Q_n\} (C_1 || \cdots || C_n) \{R_1 \cdots \cdots R_n\}
\end{array}
\]

(non-interference-of-variables)
Concurrent rules

\[
\begin{align*}
\{Q_1\} \quad C_1 \quad \{R_1\} \quad \cdots \quad & \quad \{Q_n\} \quad C_n \quad \{R_n\} \\
\{Q_1 \ast \cdots \ast Q_n\} \quad (C_1 \parallel \cdots \parallel C_n)\{R_1 \ast \cdots \ast R_n\} & \quad \text{(non-interference-of-variables)} \\
\{Q \ast I_r\} \wedge B \quad C\{R \ast I_r\} & \quad \text{(non-interference-of-variables)} \\
\{Q\} \text{with } r \text{ when } B \text{ do } C \text{ od}\{R\} & 
\end{align*}
\]
Concurrent rules

\[ \begin{align*}
\{Q_1\} & C_1 \{R_1\} \cdots \{Q_n\} C_n \{R_n\} \\
\{Q_1 \star \cdots \star Q_n\} & (C_1 \parallel \cdots \parallel C_n)\{R_1 \star \cdots \star R_n\}
\end{align*} \]

\[ \begin{align*}
\{ (Q \star I_r) \land B \} & C \{ R \star I_r \} \\
\{Q\} & \text{with } r \text{ when } B \text{ do } C \text{ od}\{R\}
\end{align*} \]

- Both proved sound by Brookes.
Conncurrency rules

\[
\begin{align*}
&\{Q_1\} C_1 \{R_1\} \cdots \{Q_n\} C_n \{R_n\} \\
\{Q_1 \star \cdots \star Q_n\} (C_1 \parallel \cdots \parallel C_n) \{R_1 \star \cdots \star R_n\} \\
&(\text{non-interference-of-variables})
\end{align*}
\]

\[
\begin{align*}
&\{(Q \star I_r) \land B\} C\{R \star I_r\} \\
&\{Q\} \text{with } r \text{ when } B \text{ do } C \text{ od}\{R\} \\
&(\text{non-interference-of-variables})
\end{align*}
\]

- Both proved sound by Brookes.
- A version of the CCR rule covers semaphores, in which $C$ is either $m := m + 1$ or $m := m - 1$. 
The ownership trick (O’Hearn)

Resource $r : \text{Vars } full, b$;

\[
\begin{align*}
& x := \text{new()} ; \\
 & \text{with } r \text{ when } \neg full \text{ do} \\
& \quad b := x ; \\
& \quad full := \text{true} \\
& \text{od}
\end{align*}
\]

\[
\begin{align*}
& \text{with } r \text{ when } full \text{ do} \\
& \quad y := b ; \\
& \quad full := \text{false} \\
& \text{od} ; \\
& \text{dispose } y
\end{align*}
\]
The ownership trick (O’Hearn)

Resource $r : \text{Vars } full, b$;

\[
\begin{align*}
\{\text{emp}\} \\
x := \text{new}();
\end{align*}
\]

with $r$ when $\neg full$ do

\[
\begin{align*}
b &:= x; \\
full &:= \text{true}
\end{align*}
\]

```
with r when ¬full do
    y := b;
    full := false
    od;
```

dispose $y$

\[
\begin{align*}
\{\text{emp}\}
\end{align*}
\]
The ownership trick (O’Hearn)

Resource $r : \text{Vars} \ full, b$;

Invariant $(\text{full} \land b \leftrightarrow \bot) \lor (\neg \text{full} \land \text{emp})$

\[
\begin{align*}
\{\text{emp}\} \\
\begin{align*}
x & := \text{new}(); \\
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& \quad b := x; \\
& \quad \text{full} := \text{true} \\
\text{od} \\
\{\text{emp}\}
\end{align*}
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\text{dispose } y \\
\{\text{emp}\}
\end{align*}
\]
The ownership trick (O’Hearn)

Resource $r : \text{Vars } full, b$;

Invariant $(full \land b \mapsto \_ ) \lor (\neg full \land \text{emp})$

```
{emp}
x := \text{new}();
{x \mapsto \_}
with r when \neg full do

\hspace{1em} b := x;

\hspace{1em} full := \text{true}

od

{emp}
```

```
{emp}
with r when full do

\hspace{1em} y := b;

\hspace{1em} full := \text{false}

od;

dispose y

{emp}
```
The ownership trick (O’Hearn)

Resource \( r : \text{Vars full, } b; \)

Invariant \((\text{full } \land b \mapsto \_ \land \land (\neg \text{full } \land \text{emp})\)

\[
\begin{align*}
\{\text{emp}\} \\
x := \text{new}(); \\
\{x \mapsto \_\} \\
\text{with } r \text{ when } \neg \text{full} \text{ do} \\
\quad \{\neg \text{full } \land \text{emp } \star x \mapsto \_\} \\
\quad b := x; \\
\quad \text{full} := \text{true}
\end{align*}
\]

\[
\begin{align*}
\text{od} \\
\{\text{emp}\}
\end{align*}
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x &:= \text{new}(); \\
\{x \leftrightarrow \_\} \\
\text{with } r \text{ when } \neg \text{full} \text{ do} \\
\ \quad \{\neg \text{full} \land \text{emp} \ast x \leftrightarrow \_\} \\
\ \quad b &:= x; \\
\ \quad \{\neg \text{full} \land \text{emp} \ast x \leftrightarrow \_ \land b = x\} \\
\ \quad \text{full} &:= \text{true} \\
\text{od} \\
\{\text{emp}\}
\end{align*}
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\begin{align*}
\{\text{emp}\} \\
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x := \text{new}(); \\
\{x \mapsto _\}\ \\
\text{with} \ r \text{ when } \neg full \text{ do} \\
\quad \{\neg full \land \text{emp} \star x \mapsto _\} \\
\quad b := x; \\
\quad \{\neg full \land \text{emp} \star x \mapsto _\land b = x\} \\
\quad full := \text{true} \\
\quad \{full \land b \mapsto _\star \text{emp}\} \\
\od \\
\{\text{emp}\}
\end{align*}
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\text{dispose} \ y \\
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b := x; \\
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o d \\
\{\text{emp}\}
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with $r$ when $full$ do

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y := b; \\
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dispose $y$
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\od \\
\{ \text{emp} \}
\end{align*}
\]

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\begin{align*}
\{ \text{emp} \} \\
\text{with } r \text{ when } full \text{ do} \\
& \quad \{ full \land b \mapsto _ \star \text{emp} \} \\
& \quad y := b; \\
& \quad \{ full \land b \mapsto _ \star \text{emp} \land y = b \} \\
& \quad full := \text{false} \\
& \quad \text{od;} \\
& \quad \text{dispose } y \\
\{ \text{emp} \}
\end{align*}
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\quad b := x; \\
\quad \{\neg \textit{full} \land \textit{emp} \ast x \mapsto \_ \land b = x\} \\
\quad \textit{full} := \text{true} \\
\quad \{\textit{full} \land b \mapsto \_ \ast \textit{emp}\} \\
\od \\
\{\textit{emp}\}
\end{align*}$

$\begin{align*}
\{\textit{emp}\} \\
\text{with } r \text{ when } \textit{full} \text{ do} \\
\quad \{\textit{full} \land b \mapsto \_ \ast \textit{emp}\} \\
\quad y := b; \\
\quad \{\textit{full} \land b \mapsto \_ \ast \textit{emp} \land y = b\} \\
\quad \textit{full} := \text{false} \\
\quad \{\neg \textit{full} \land \textit{emp} \ast y \mapsto \_\} \\
\od; \\
\{y \mapsto \_\} \\
\text{dispose } y \\
\{\textit{emp}\}
\end{align*}$
Part I

Passivity, sharing, accounting
Passivity

Passivity is a property of a program and a resource: the program doesn’t change the contents of the resource.

We want to specify passivity by specifying a read-only resource.

We require that a program, given a read-only resource, cannot change its contents.
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- We want to specify passivity by specifying a read-only resource.
- We require that a program, given a read-only resource, cannot change its contents.
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Concurrent read permissions must be separable, because of the concurrency rule.
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Splitting and sharing

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- Total permission $E \mapsto E'$, given by new, allows read/write/dispose.
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- Total permission $E \mapsto E'$, given by new, allows read/write/dispose.
- Concurrent read permissions must be ($\star$) separable, because of the concurrency rule.
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Accounting

- Splitting into multiple read permissions is easy.
- To write or dispose we have to know when we have all the read permissions back.
- A program which doesn’t keep account leaks resource.
Boyland’s suggestion: \( \frac{1}{2} + \frac{1}{2} = 1 \)
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- Boyland (Wisconsin) developed a means of permission accounting in disjoint concurrency, dealing with variables and heap locations.
- He associates a number \( z \) with each permission: \( z = 1 \) total; \( 0 < z < 1 \) read-only.
- Fractional permissions are specification-only (cf. types).
- In practice the arithmetic is very easy: fractions are simpler to use than (e.g.) sets of binary trees.
- The magnitude of non-integral fractions doesn’t matter, except as a matter of accounting.
A fractional model
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- Heaps are now partial maps from Nat to (int, fraction). (Previously Nat to int.)
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- Heaps are now partial maps from Nat to (int, fraction).  
  (Previously Nat to int.)
- A simpler model – just read / total permissions – fails to account and doesn’t have the frame property.
Proof theory
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\[ E \xleftrightarrow{z} E' \quad \Rightarrow \quad 0 < z \leq 1 \]

\[ E \xrightarrow{z+z'} E' \land z > 0 \land z' > 0 \quad \iff \quad E \xrightarrow{z} E' \ast E \xrightarrow{z'} E' \]
Proof theory

\[ E \mapsto_{z} E' \quad \Rightarrow \quad 0 < z \leq 1 \]

\[ E \mapsto_{z+z'} E' \land z > 0 \land z' > 0 \quad \iff \quad E \mapsto_{z} E' \ast E \mapsto_{z'} E' \]

\[
\begin{align*}
\{ & R^x_E \} & x := E & \{ R \} \\
\{ E' \mapsto_1 - \} & [E'] := E & \{ E' \mapsto_1 E \} \\
\{ E' \mapsto z E \} & x := [E'] & \{ E' \mapsto z E \land x = E \} & (x \text{ not free in } E, E') \\
\{ \text{emp} \} & x := \text{new}(E) & \{ x \mapsto_1 E \} \\
\{ E \mapsto_1 - \} & & \text{dispose } E & \{ \text{emp} \}
\end{align*}
\]
Proof theory

\[ E \mapsto_{z} E' \quad \Rightarrow \quad 0 < z \leq 1 \]
\[ E \mapsto_{z + z'} E' \land z > 0 \land z' > 0 \quad \iff \quad E \mapsto_{z} E' \ast E \mapsto_{z'} E' \]

\[
\begin{align*}
\{ R_E^x \} & \quad x := E \\
\{ E' \mapsto_{1} - \} & \quad [E'] := E \\
\{ E' \mapsto_{z} E \} & \quad x := [E'] \\
\{ \text{emp} \} & \quad x := \text{new}(E) \\
\{ E \mapsto_{1} - \} & \quad \text{dispose } E
\end{align*}
\]

\[
\begin{align*}
\{ R \} & \quad E' \mapsto_{1} E \\
\{ E' \mapsto_{z} E \land x = E \} & \quad (x \text{ not free in } E, E') \\
\{ x \mapsto_{1} E \} & \quad \text{dispose } E
\end{align*}
\]

- Not (yet) proved sound by Brookes. (But surely ...)

\[ \triangleright \]

\[
\begin{align*}
\{ \text{emp} \} & \quad x := \text{new}(E) \\
\{ E \mapsto_{1} - \} & \quad \text{dispose } E
\end{align*}
\]
Proof

\[
\{ \text{emp} \} \\
x := \text{new}();
\]

\[
[x] := 1;
\]

\[
\begin{pmatrix}
\text{y := } [x] \\
\text{z := } [x] + 1
\end{pmatrix}
\]

dispose x

\[
\{ \text{emp} \land y = 1 \land z = 2 \}
\]
Proof

\{\text{emp}\}
\begin{align*}
x & := \text{new}(); \\
\{x \mapsto -1\} & \\
[x] & := 1;
\end{align*}
\begin{pmatrix}
y := [x] \\
z := [x] + 1
\end{pmatrix};
dispose x
\{\text{emp} \land y = 1 \land z = 2\}

That is exactly how hard it is to use fractional permissions.
Proof

\begin{align*}
\{ \text{emp} \} \\
x &:= \text{new}(); \\
\{ x \mapsto 1 \} \\
[x] &:= 1; \\
\{ x \mapsto 1 \} \\
\left( \begin{array}{l}
\{ y := [x] \} \\
\{ z := [x] + 1 \}
\end{array} \right); \\
\text{dispose } x \\
\{ \text{emp} \land y = 1 \land z = 2 \}
\end{align*}

That is exactly how hard it is to use fractional permissions.
**Proof**

\[
\begin{align*}
\{ \text{emp} \} \\
x &:= \text{new}(); \\
\{ x \mapsto 1 \} \\
[x] &:= 1; \\
\{ x \mapsto 1 \} &\cdot \{ x \mapsto 0.5 \ \ast \ x \mapsto 0.5 \ 1 \} \\
\begin{pmatrix}
y := [x] \\
z := [x] + 1
\end{pmatrix}
\end{align*}
\]

dispose \ x
\{ \text{emp} \ \land \ y = 1 \ \land \ z = 2 \}
Proof

\[
\begin{align*}
\{\text{emp}\} \\
& x := \text{new}(); \\
& \{x \mapsto 1 \} \\
& [x] := 1; \\
& \{x \mapsto 1\} \cdot \{x \overset{0.5}{\mapsto} 1 \ast x \overset{0.5}{\mapsto} 1\} \\
& \begin{cases} \\
& \{x \overset{0.5}{\mapsto} 1\} \\
y := [x] \\
& \{x \overset{0.5}{\mapsto} 1\} \\
z := [x] + 1 \\
\end{cases}
\end{align*}
\]

dispose \(x\)

\[
\{\text{emp} \land y = 1 \land z = 2\}
\]
**Proof**

\[
\begin{align*}
\{ \text{emp} \} \\
x &:= \text{new}(); \\
\{ x \mapsto 1 \} \\
x &:= 1; \\
\{ x \mapsto 1 \} &:: \{ x \mapsto 0.5 \} \times \{ x \mapsto 0.5 \} \\
\begin{pmatrix}
\{ x \mapsto 0.5 \} \\
y := [x] \\
\{ x \mapsto 0.5 \} \text{ and } y = 1
\end{pmatrix} &\parallel \\
\begin{pmatrix}
\{ x \mapsto 0.5 \} \\
z := [x] + 1
\end{pmatrix};
\end{align*}
\]

dispose x

\[
\{ \text{emp} \wedge y = 1 \wedge z = 2 \}
\]
Proof

\[
\begin{align*}
\{\text{emp}\} \\
{x} &:= \text{new}(); \\
\{x \mapsto 1\} \\
[x] &:= 1; \\
\{x \mapsto 1\} &\cdot \{x \mapsto 0.5 \ast x \mapsto 0.5 \mapsto 1\} \\
\begin{pmatrix}
\{x \mapsto 0.5 \mapsto 1\} \\
y := [x] \\
\{x \mapsto 0.5 \mapsto 1 \land y = 1\}
\end{pmatrix} &\parallel \\
\{x \mapsto 0.5 \mapsto 1\} \\
z := [x] + 1 \\
\{x \mapsto 0.5 \mapsto 1 \land z = 2\}
\end{align*}
\]

dispose \(x\) \\
\{\text{emp} \land y = 1 \land z = 2\}
Proof

\{
\text{emp}
\}

\begin{align*}
x & := \text{new}(); \\
\{x \mapsto 1\} & \\
[x] & := 1; \\
\{x \mapsto 1\} : & \{x \mapsto 0.5 \land x \mapsto 0.5 \land 1\}; \\
\begin{pmatrix}
\{x \mapsto 0.5 \land 1\} & \{x \mapsto 0.5 \land 1\} \\
y & := [x] \\
z & := [x] + 1
\end{pmatrix}; \\
\begin{align*}
\{x \mapsto 0.5 \land y = 1\} & \ | \ \{x \mapsto 0.5 \land z = 2\}; \\
\{x \mapsto 0.5 \land y = 1\} & \ | \ \{x \mapsto 0.5 \land z = 2\}
\end{align*}
\text{dispose } x

\begin{align*}
\{\text{emp} \land y = 1 \land z = 2\}
\end{align*}
Proof

\{\mathbf{emp}\}
\begin{align*}
x &:= \text{new}();
\{x \mapsto 1\}
\end{align*}

\[x := 1;\]
\begin{align*}
\{x \mapsto 1\} &:: \{x \mapsto 0.5 \ast x \mapsto 0.5 \mapsto 1\}
\end{align*}

\[
\begin{array}{c}
\{x \mapsto 0.5 \mapsto 1\} \parallel \{x \mapsto 0.5 \mapsto 1\} \\
y := [x] \parallel z := [x] + 1
\end{array}
\]

\[
\begin{array}{c}
\{x \mapsto 0.5 \mapsto 1 \land y = 1\} \parallel \{x \mapsto 0.5 \mapsto 1 \land z = 2\}
\end{array}
\]

\[
\begin{align*}
\{(x \mapsto 0.5 \mapsto 1 \land y = 1) \ast (x \mapsto 0.5 \mapsto 1 \land z = 2)\} &:: \{x \mapsto 1 \land y = 1 \land z = 2\}
\end{align*}
\]

dispose x
\begin{align*}
\{\mathbf{emp} \land y = 1 \land z = 2\}
\end{align*}

That is exactly how hard it is to use fractional permissions.
Proof

\[
\begin{align*}
\{ \text{emp} \}
\end{align*}
\]

\[
x := \text{new}();
\]

\[
\begin{align*}
\{ x \mapsto 1 \} \\
x := 1;
\end{align*}
\]

\[
\begin{align*}
\{ x \mapsto 1 \} : \{ x \mapsto 0.5 \} \ast x \mapsto 0.5 ;
\end{align*}
\]

\[
\begin{align*}
\{ y := [x] \} \quad \{ z := [x] + 1 \}
\end{align*}
\]

\[
\begin{align*}
\{ x \mapsto 0.5 ; x \mapsto 0.5 \} ;
\end{align*}
\]

\[
\begin{align*}
\{ (x \mapsto 0.5 \ast y = 1) \ast (x \mapsto 0.5 \ast z = 2) \} : \{ x \mapsto 1 \ast y = 1 \ast z = 2 \}
\end{align*}
\]

\[
\text{dispose } x \\
\{ \text{emp} \ast y = 1 \ast z = 2 \}
\]

That is exactly how hard it is to use fractional permissions.
UnProof

\[
\begin{align*}
\{\text{emp}\} \\
x & := \text{new}(); \\
\{x \mapsto 1\} \\
[x] & := 1; \\
\{x \mapsto 1\} \cdot \{x \mapsto 0.5 \rightarrow 1 \} \times \{x \mapsto 0.5 \rightarrow 1\} \\
y & := [x]; \\
disable x \\
\{x \mapsto 0.5 \rightarrow 1\} \\
[x] & := 2; \\
z & := [x] + 1 \\
\{x \mapsto 0.5 \rightarrow 1\} \\
[x] & := y + z
\end{align*}
\]
\[
\begin{align*}
\{\text{emp}\} \\
& \quad x := \text{new}(); \\
& \quad \{x \mapsto -\} \\
& \quad [x] := 1; \\
& \quad \{x \mapsto 1\} \models \{x \mapsto 0.5 \mapsto 1 \ast x \mapsto 0.5 \mapsto 1\} \\
& \quad \begin{cases}
\{x \mapsto 0.5 \mapsto 1\} \\
y := [x]; \\
\{x \mapsto 0.5 \mapsto 1 \land y = 1\} \quad \| \quad \{x \mapsto 0.5 \mapsto 1\}
\end{cases}
\quad \begin{cases}
\text{dispose } x \\
[x] := 2; \\
\{x \mapsto 0.5 \mapsto 1\}
\end{cases}
\quad \begin{cases}
z := [x] + 1
\end{cases}
\end{align*}
\]

\[
[x] := y + z
\]
UnProof

\{
\texttt{emp}\}

\begin{align*}
x & := \text{new}() ; \\
\{ x \mapsto \_ \} & \\
[x] & := 1 ; \\
\{ x \mapsto 1 \} & : \{ x \mapsto 0.5 \ 1 \ast x \mapsto 0.5 \ 1 \} \\
\begin{pmatrix} 
\{ x \mapsto 0.5 \ 1 \} \\
y & := [x] ; \\
\{ x \mapsto 0.5 \ 1 \land y = 1 \} \\
dispose \ x \\
\{ ?? \} \\
\end{pmatrix} & \quad \| \quad \\
\begin{pmatrix}
\{ x \mapsto 0.5 \ 1 \} \\
[x] & := 2 ; \\
\{ x \mapsto 0.5 \ 1 \} \\
z & := [x] + 1 \\
\end{pmatrix} \\
\end{align*}

[x] := y + z
\begin{equation*}
\{ \text{emp} \}
\end{equation*}
\begin{align*}
x & := \text{new}(); \\
\begin{cases}
x & \mapsto - \\
\end{cases} \\
[x] & := 1; \\
\begin{cases}
x & \mapsto 1 \\
\end{cases} & \cdot \\
\begin{cases}
x & \mapsto 0.5 1 \\
\end{cases} & \ast \\
\begin{cases}
x & \mapsto 0.5 1 \\
\end{cases}
\end{align*}
\begin{align*}
y & := [x]; \\
\begin{cases}
x & \mapsto 0.5 1 \land y = 1 \\
\end{cases} & \text{dispose } x \\
\begin{cases}
?? \\
\end{cases}
\end{align*}
\begin{align*}
[x] & := 2; \\
\begin{cases}
?? \\
\end{cases} & \text{dispose } x \\
\begin{cases}
?? \\
\end{cases} & z := [x] + 1
\end{align*}
\begin{align*}
[x] & := y + z
\end{align*}
\[
\begin{align*}
\{ \text{emp} \} \\
x &:= \text{new}(); \\
\{ x \mapsto 1 \} \\
[x] &:= 1; \\
\{ x \mapsto 1 \} &:: \{ x \mapsto 0.5 \ (1 + x) \mapsto 0.5 \ 1 \} \\
\begin{pmatrix}
\{ x \mapsto 0.5 \ 1 \} \\
y &:= [x]; \\
\{ x \mapsto 0.5 \ (1 + y = 1) \} \\
\text{dispose } x \\
\{ ?? \} \\
\{ ?? \}
\end{pmatrix} &||
\begin{pmatrix}
\{ x \mapsto 0.5 \ 1 \} \\
[x] &:= 2; \\
\{ ?? \} \\
\{ ?? \}
\end{pmatrix} \\
[x] &:= y + z
\end{align*}
\]
Passivity and fractions

Termination Monotonicity: if $C$ must terminate normally in $h$ and $h \star h'$ is defined, then $C$ must terminate normally in $h \star h'$.
Passivity and fractions

Termination Monotonicity: if $C$ must terminate normally in $h$ and $h \star h'$ is defined, then $C$ must terminate normally in $h \star h'$.

- We can prove termination monotonicity for all commands in our language.
Passivity and fractions

Termination Monotonicity: if $C$ must terminate normally in $h$ and $h \star h'$ is defined, then $C$ must terminate normally in $h \star h'$.

- We can prove termination monotonicity for all commands in our language.
- Suppose $\left\{10 \xrightarrow{0.5} N\right\} C \left\{10 \xrightarrow{0.5} N + 1\right\}$, and it terminates.
Passivity and fractions

Termination Monotonicity: if $C$ must terminate normally in $h$ and $h \ast h'$ is defined, then $C$ must terminate normally in $h \ast h'$.

- We can prove termination monotonicity for all commands in our language.
- Suppose $\{10 \mathrel{\xrightarrow{0.5}} N\} C \{10 \mathrel{\xrightarrow{0.5}} N + 1\}$, and it terminates.
- Then (frame rule)

  $\vdash$

  $\frac{\{10 \mathrel{\xrightarrow{0.5}} N\} C \{10 \mathrel{\xrightarrow{0.5}} N + 1\}}{\{10 \mathrel{\xrightarrow{0.5}} N \ast 10 \mathrel{\xrightarrow{0.5}} N\} C \{10 \mathrel{\xrightarrow{0.5}} N \ast 10 \mathrel{\xrightarrow{0.5}} N + 1\}}$
Passivity and fractions

Termination Monotonicity: if $C$ must terminate normally in $h$ and $h \ast h'$ is defined, then $C$ must terminate normally in $h \ast h'$.

- We can prove termination monotonicity for all commands in our language.
- Suppose $\left\{ 10 \overset{0.5}{\rightarrow} N \right\} C \left\{ 10 \overset{0.5}{\rightarrow} N + 1 \right\}$, and it terminates.
- Then (frame rule)

$$\begin{align*}
\left\{ 10 \overset{0.5}{\rightarrow} N \right\} C \left\{ 10 \overset{0.5}{\rightarrow} N + 1 \right\} \\
\left\{ 10 \overset{0.5}{\rightarrow} N \ast 10 \overset{0.5}{\rightarrow} N \right\} C \left\{ 10 \overset{0.5}{\rightarrow} N \ast 10 \overset{0.5}{\rightarrow} N + 1 \right\}
\end{align*}$$

- i.e. it won’t terminate in $10 \overset{1.0}{\rightarrow} N$. 

Therefore $C$ is in our language. Thus we have passivity!
Passivity and fractions

**Termination Monotonicity:** if $C$ must terminate normally in $h$ and $h \ast h'$ is defined, then $C$ must terminate normally in $h \ast h'$.

- We can prove termination monotonicity for all commands in our language.
- Suppose $\{10 \xrightarrow{0.5} N\} C \{10 \xrightarrow{0.5} N + 1\}$, and it terminates.
- Then (frame rule)

\[
\begin{align*}
\{10 \xrightarrow{0.5} N\} C \{10 \xrightarrow{0.5} N + 1\} \\
\{10 \xrightarrow{0.5} N \ast 10 \xrightarrow{0.5} N\} C \{10 \xrightarrow{0.5} N \ast 10 \xrightarrow{0.5} N + 1\}
\end{align*}
\]
- i.e. it won’t terminate in $10 \xrightarrow{1.0} N$.
- Therefore $C$ isn’t in our language.
Passivity and fractions

Termination Monotonicity: if $C$ must terminate normally in $h$ and $h \times h'$ is defined, then $C$ must terminate normally in $h \times h'$.

- We can prove termination monotonicity for all commands in our language.
- Suppose $\left\{ 10 \xrightarrow{0.5} N \right\}C\left\{ 10 \xrightarrow{0.5} N + 1 \right\}$, and it terminates.
- Then (frame rule)

\[
\left\{ 10 \xrightarrow{0.5} N \right\}C\left\{ 10 \xrightarrow{0.5} N + 1 \right\} \\
\left\{ 10 \xrightarrow{0.5} N \times 10 \xrightarrow{0.5} N \right\}C\left\{ 10 \xrightarrow{0.5} N \times 10 \xrightarrow{0.5} N + 1 \right\}
\]

- i.e. it won’t terminate in $10 \xrightarrow{1.0} N$.
- Therefore $C$ isn’t in our language.
- Thus we have passivity!
Part II

Counting permissions
Permission counting

Some programs naturally weigh out permissions to their child threads: e.g. parallel tree-copy, parallel tree-rewriting (see proceedings).

Some programs count permissions: e.g. pipeline multicasting, readers-and-writers.

Permission counting is not specification-only.
Permission counting

- Some programs naturally weigh out permissions to their child threads: e.g. parallel tree-copy, parallel tree-rewriting (see proceedings).
Permission counting

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- Some programs count permissions: e.g. pipeline multicasting, readers-and-writers.
Permission counting

- Some programs naturally weigh out permissions to their child threads: e.g. parallel tree-copy, parallel tree-rewriting (see proceedings).
- Some programs count permissions: e.g. pipeline multicasting, readers-and-writers.
- Permission counting is not specification-only.
Readers and writers (CCR version)
Readers and writers (CCR version)

with \textit{read} when true do
  if \textit{count} = 0 then \texttt{P(\texttt{write})}
  else skip fi;
  \textit{count} + := 1
od;

... reading happens here ...

with \textit{read} when true do
  \textit{count} − := 1;
  if \textit{count} = 0 then \texttt{V(\texttt{write})}
  else skip fi
od

\texttt{P(\texttt{write});}

... writing happens here ...

\texttt{V(\texttt{write})}
Readers and writers (CCR version)

\{	extit{emp}\}\}

\textbf{with} \textit{read} \textbf{when} \textbf{true} \textbf{do}
\begin{align*}
\textbf{if} \hspace{1em} \textit{count} &= 0 \textbf{then} \textit{P(write)} \\
&\hspace{2em} \text{else skip fi;}
\end{align*}
\begin{align*}
\textit{count} + &\hspace{1em} := 1 \\
\textbf{od;}
\end{align*}
\begin{align*}
\begin{array}{c}
\{z \mapsto N\}
\end{array}
\end{align*}
\textbf{... reading happens here ...}

\textbf{with} \textit{read} \textbf{when} \textbf{true} \textbf{do}
\begin{align*}
\textit{count} - &\hspace{1em} := 1; \\
\textbf{if} \hspace{1em} \textit{count} &= 0 \textbf{then} \textit{V(write)} \\
&\hspace{2em} \text{else skip fi}
\end{align*}
\begin{align*}
\textbf{od}
\end{align*}
\begin{align*}
\textit{P(write);} \\
\textbf{... writing happens here ...}
\end{align*}
\begin{align*}
\textit{V(write)}
\end{align*}
Readers and writers (CCR version)

\[
\{ \text{emp} \} \\
\text{with } \text{read \ when \ true \ do} \\
\quad \text{if } \text{count} = 0 \text{ then } \text{P(write)} \\
\quad \quad \text{else skip fi}; \\
\quad \text{count} + := 1 \\
\text{od;} \\
\{ z \mapsto N \} \\
\text{... reading happens here ...} \\
\{ z \mapsto N \} \\
\text{with } \text{read \ when \ true \ do} \\
\quad \text{count} - := 1; \\
\quad \text{if } \text{count} = 0 \text{ then } \text{V(write)} \\
\quad \quad \text{else skip fi} \\
\text{od} \\
\{ \text{emp} \} \\
\]

\[
\text{P(write);} \\
\text{... writing happens here ...} \\
\text{V(write)}
\]
Readers and writers (CCR version)

\{	ext{emp}\}
with read when true do
   if count = 0 then P(write)
       else skip fi;
   count+ := 1
od;
\{z \mapsto N\}
   ... reading happens here ...
\{z \mapsto N\}
with read when true do
   count− := 1;
   if count = 0 then V(write)
       else skip fi
od
\{\text{emp}\}

\{\text{emp}\}
\{z \mapsto M\}
   ... writing happens here ...
\{z \mapsto M'\}
V(write)
Readers and writers (CCR version)

\{\text{emp}\}
with \text{read} \text{ when true do}
  \text{if } count = 0 \text{ then } \text{P}(\text{write})
  \text{else skip fi;}
  count + := 1
od;
\{z \mapsto N\}
... reading happens here ...
\{z \mapsto N\}
with \text{read} \text{ when true do}
  count - := 1;
  \text{if } count = 0 \text{ then } \text{V}(\text{write})
  \text{else skip fi}
od
\{\text{emp}\}
A counting model

Heaps are partial maps from Nat to (int, permission).

Permissions are

- $-n$ (read permissions), or
- $+n$ (a "block" from which $n$ read permissions have been "flaked").

0 is total permission.

$E_i \mapsto E' \triangleright E_j \mapsto E' = \begin{cases} 
\text{undefined} & (i \geq 0 \land j \geq 0 \land i + j < 0) \\
\text{undefined} & (i \geq 0 \lor j \geq 0) \\
E_i + j \mapsto E' & \text{otherwise} 
\end{cases}$

$E \rightarrow E'$ is a notational convenience for $E \rightarrow E' - 1$.

We have passivity (same proof as before).
A counting model

- Heaps are partial maps from Nat to (int, permission).
A counting model

- Heaps are partial maps from Nat to (int, permission).
- Permissions are $-n$ ($n$ read permissions), or $+n$ (a “block” from which $n$ read permissions have been “flaked”).
A counting model

- Heaps are partial maps from Nat to (int, permission).
- Permissions are \(-n\) \((n\text{ read permissions})\), or \(+n\) (a “block” from which \(n\) read permissions have been “flaked”).
- 0 is total permission.
A counting model

- Heaps are partial maps from Nat to (int, permission).
- Permissions are $-n$ ($n$ read permissions), or $+n$ (a “block” from which $n$ read permissions have been “flaked”).
- 0 is total permission.

\[
E \leftrightarrow^i E' \star E \leftrightarrow^j E' = \begin{cases} 
\text{undefined} & i \geq 0 \land j \geq 0 \\
\text{undefined} & (i \geq 0 \lor j \geq 0) \land i + j < 0 \\
E \leftrightarrow^{i+j} E' & \text{otherwise}
\end{cases}
\]
A counting model

- Heaps are partial maps from Nat to (int, permission).
- Permissions are $-n$ ($n$ read permissions), or $+n$ (a “block” from which $n$ read permissions have been “flaked”).
- $0$ is total permission.
- $E \rightarrow E' \star E \leftarrow E' = \begin{cases} 
  \text{undefined} & i \geq 0 \land j \geq 0 \\
  \text{undefined} & (i \geq 0 \lor j \geq 0) \land i + j < 0 \\
  E \leftarrow^{i+j} E' & \text{otherwise} 
\end{cases}$
- $E \rightarrow E'$ is a notational convenience for $E \leftarrow^{-1} E'$. 
A counting model

- Heaps are partial maps from Nat to (int, permission).
- Permissions are $-n$ ($n$ read permissions), or $+n$ (a “block” from which $n$ read permissions have been “flaked”).
- 0 is total permission.

$$E \overset{i}{\mapsto} E' \star E \overset{j}{\mapsto} E' = \begin{cases} 
\text{undefined} & i \geq 0 \land j \geq 0 \\
\text{undefined} & (i \geq 0 \lor j \geq 0) \land i + j < 0 \\
E \overset{i+j}{\mapsto} E' & \text{otherwise}
\end{cases}$$

- $E \mapsto E'$ is a notational convenience for $E \overset{-1}{\mapsto} E'$.
- We have passivity (same proof as before).
Proof theory
Proof theory

\[ E \xleftarrow{n} E' \Rightarrow n \geq 0 \]

\[ E \xrightarrow{n} E' \iff E \xrightarrow{n+1} E' \star E \rightarrow E' \]
Proof theory

\[ E \xrightarrow{n} E' \Rightarrow n \geq 0 \]
\[ E \xrightarrow{n} E' \iff E \xrightarrow{n+1} E' \star E \Rightarrow E' \]

\[
\begin{align*}
\{ R_E^\ell \} & \quad x := E & \{ R \} \\
\{ E' \xrightarrow{0} - \} & \quad [x] := E & \{ E' \xrightarrow{0} E \} \\
\{ E' \Rightarrow E \} & \quad x := [E'] & \{ E' \Rightarrow E \land x = E \} & (x \text{ not free in } E, E') \\
\{ \text{emp} \} & \quad x := \text{new}(E) & \{ x \xrightarrow{0} E \} \\
\{ E \xrightarrow{0} - \} & \quad \text{dispose } E & \{ \text{emp} \}
\end{align*}
\]
Resource safety proof

\[\text{write} : \text{if } \text{write} = 0 \text{ then } \text{emp} \text{ else } z \mapsto^0 N \text{ fi}\]
\[\text{read} : \text{if } \text{count} = 0 \text{ then } \text{emp} \text{ else } z \mapsto^{\text{count}} N \text{ fi}\]

\{\text{emp}\}

with \text{read} when true do

\[
\begin{align*}
\text{if } \text{count} = 0 \text{ then } & \quad \text{P}(\text{write}) \\
\text{else } & \quad \text{skip} \\
\text{fi;}
\end{align*}
\]

\[
\text{count} + := 1
\]

od

\{z \mapsto^* N\}
Resource safety proof

\(\text{write} : \text{if } write = 0 \text{ then } \text{emp} \text{ else } z \stackrel{0}{\rightarrow} N \text{ fi}\)

\(\text{read} : \text{if } count = 0 \text{ then } \text{emp} \text{ else } z \stackrel{count}{\rightarrow} N \text{ fi}\)

\(\{\text{emp}\}\)

with read when true do

\(\{\text{if } count = 0 \text{ then } \text{emp} \text{ else } z \stackrel{count}{\rightarrow} N \text{ fi } \ast \text{emp}\}\)

if \(\text{count} = 0\) then \(\text{P(write)}\)

\(\text{else skip}\)

\(\text{fi;}\)

\(\text{count} + := 1\)

od

\(\{z \rightarrow N\}\)
Resource safety proof

\[\text{write} : \text{if } write = 0 \text{ then } \textbf{emp} \text{ else } z \xrightarrow{0} N \text{ fi}\]

\[\text{read} : \text{if } count = 0 \text{ then } \textbf{emp} \text{ else } z \xrightarrow{\text{count}} N \text{ fi}\]

\{\textbf{emp}\}
with \text{read} when true do

\{\text{if } count = 0 \text{ then } \textbf{emp} \text{ else } z \xrightarrow{\text{count}} N \text{ fi } \ast \textbf{emp}\}\]

if \( count = 0 \) then \{\textbf{emp}\} \mathbf{P}(\text{write})
else \{z \xrightarrow{\text{count}} N\} \text{ skip}
fi;

\( count + := 1 \)

od

\{z \rightarrow N\}
Resource safety proof

\text{write} : \text{if } \text{write} = 0 \text{ then } \text{emp} \text{ else } z \xrightarrow{0} N \text{ fi}

\text{read} : \text{if } \text{count} = 0 \text{ then } \text{emp} \text{ else } z \xrightarrow{\text{count}} N \text{ fi}

\{\text{emp}\}

\text{with read when true do}

\{ \text{if } \text{count} = 0 \text{ then } \text{emp} \text{ else } z \xrightarrow{\text{count}} N \text{ fi } \} \star \{\text{emp}\}

\text{if } \text{count} = 0 \text{ then}

\{\text{emp}\} \ \text{P(\text{write})} \ \{z \xrightarrow{0} N\}

\text{else} \ \{z \xrightarrow{\text{count}} N\} \ \text{skip}

\text{fi;}

\text{count} + := 1

\text{od}

\{z \xrightarrow{} N\}
**Resource safety proof**

\( \text{write} : \text{if } \text{write} = 0 \text{ then } \text{emp} \text{ else } z \leftarrow^0 N \text{ fi} \)

\( \text{read} : \text{if } \text{count} = 0 \text{ then } \text{emp} \text{ else } z \leftarrow^\text{count} N \text{ fi} \)

\{ \text{emp} \}

with \text{read} when true do

\{ \text{if } \text{count} = 0 \text{ then } \text{emp} \text{ else } z \leftarrow^\text{count} N \text{ fi } \star \text{emp} \}

\{ \text{if } \text{count} = 0 \text{ then } \text{emp} \text{ P(}\text{write}) \text{ else } \}

\{ z \leftarrow^\text{count} N \}

skip \{ z \leftarrow^\text{count} N \}

\text{fi;}

\text{count}+ := 1

\{ z \rightarrow N \}
Resource safety proof

\[\text{write} : \text{if} \ write = 0 \text{ then emp else } z \mapsto 0 \rightarrow N \text{ fi}\]
\[\text{read} : \text{if} \ count = 0 \text{ then emp else } z \mapsto count \rightarrow N \text{ fi}\]

\{\text{emp}\}

with \ read \ when \ true \ do
\{\text{if} \ count = 0 \text{ then emp else } z \mapsto count \rightarrow N \text{ fi} \ast \text{emp}\}
if \ count = 0 \text{ then } \{\text{emp}\} \ P(\text{write}) \ \{z \mapsto 0 \rightarrow N\}\]
else \ \{z \mapsto count \rightarrow N\} \ \text{skip} \ \{z \mapsto count \rightarrow N\}\]
fi;
\{z \mapsto count \rightarrow N\}
\text{count} + := 1

\od\]
\{z \mapsto \rightarrow N\}
Resource safety proof

\[ \text{write} : \text{if } \text{write} = 0 \text{ then } \text{emp} \text{ else } z \mapsto 0 N \text{ fi} \]
\[ \text{read} : \text{if } \text{count} = 0 \text{ then } \text{emp} \text{ else } z \mapsto \text{count} N \text{ fi} \]

\{ \text{emp} \}
with \text{read} \ when \ true \ do

\{ \text{if } \text{count} = 0 \text{ then } \text{emp} \text{ else } z \mapsto \text{count} N \text{ fi } \star \text{emp} \}

if \text{count} = 0 \text{ then}

\{ \text{emp} \} \ \text{P(write)} \ \{ z \mapsto 0 N \}

else

\{ z \mapsto \text{count} N \} \ \text{skip} \ \{ z \mapsto \text{count} N \}

fi;

\{ z \mapsto \text{count} N \}

\text{count} + := 1

\{ z \mapsto \text{count} - 1 N \}

od

\{ z \mapsto N \}
Resource safety proof

\textit{write} : \textit{if} \ \textit{write} = 0 \ \textit{then} \ \texttt{emp} \ \textit{else} \ z \xrightleftharpoons{0} N \ \textit{fi}

\textit{read} : \textit{if} \ \textit{count} = 0 \ \textit{then} \ \texttt{emp} \ \textit{else} \ z \xrightarrow{\textit{count}} N \ \textit{fi}

\{\texttt{emp}\}

\textit{with read} \ \textit{when} \ \textit{true} \ \textit{do}

\{\textit{if} \ \textit{count} = 0 \ \textit{then} \ \texttt{emp} \ \textit{else} \ z \xrightarrow{\textit{count}} N \ \textit{fi \ \ast \ emp}\}

\textit{if} \ \textit{count} = 0 \ \textit{then}

\{\texttt{emp}\} \ \textit{P(\textit{write})} \ \{z \xrightarrow{0} N\}

\textit{else}

\{z \xrightarrow{\textit{count}} N\} \ \textit{skip} \ \{z \xrightarrow{\textit{count}} N\}

\textit{fi;}

\{z \xrightarrow{\textit{count}} N\}

\textit{count} + := 1

\{z \xrightarrow{\textit{count} - 1} N\} \ \ldots \ \{z \xrightarrow{\textit{count}} N \ast z \xrightarrow{} N\}

\textit{od}

\{z \xrightarrow{} N\}
Something you may have missed ...
Something you may have missed …

- Readers-and-writers has several distinct parts: reader prologue, reader action, reader epilogue, writer P, writer action, writer V.
Something you may have missed ... 

- Readers-and-writers has several distinct parts: reader prologue, reader action, reader epilogue, writer P, writer action, writer V.

\[
\text{prologue; prologue; prologue; }
\begin{pmatrix}
\text{reader}_1; \\
\text{epilogue} \\
\text{reader}_2 \\
\text{reader}_3
\end{pmatrix}; \\
\text{epilogue; reader}_4; \text{epilogue}
\]
Something you may have missed ...

- Readers-and-writers has several distinct parts: reader prologue, reader action, reader epilogue, writer P, writer action, writer V.

\[
\text{prologue}; \text{prologue}; \text{prologue};
\]
\[
\left( \text{reader}_1; \text{reader}_2 \parallel \text{reader}_3 \right);
\]
\[
\text{epilogue; reader}_4; \text{epilogue}
\]
\[
\text{P(write); writer}_1; \left( \text{reader}_5 \parallel \text{reader}_6 \right); \text{writer}_2; \text{V(write)}
\]
Something you may have missed ...

- Readers-and-writers has several distinct parts: reader prologue, reader action, reader epilogue, writer P, writer action, writer V.

\[
\text{prologue} \mid \text{prologue} \mid \text{prologue};
\]

\[
\left( \text{reader}_1 \mid \text{reader}_2 \mid \text{reader}_3 \right)
\]

\[
\text{epilogue} \mid \text{epilogue}; \text{reader}_4 \mid \text{epilogue}
\]

- \( P(\text{write}) \mid \text{writer}_1 \mid \left( \text{reader}_5 \mid \text{reader}_6 \right) \mid \text{writer}_2 \mid V(\text{write}) \)

- No more critical sections! – now that’s resourcing!
We do have some unsolved problems ...
We do have some unsolved problems ...

\[
\begin{align*}
\text{ztree } z \text{ nil Empty } & \triangleq \textbf{emp} \\
\text{ztree } z \ t \ (\text{Tip } \alpha) & \triangleq t \mapsto 0, \alpha \\
\text{ztree } z \ t \ (\text{Node } \lambda \ \rho) & \triangleq \exists l, r \cdot (t \mapsto 1, l, r \star \text{ztree } z \ l \ \lambda \star \text{ztree } z \ r \ \rho)
\end{align*}
\]
We do have some unsolved problems ...

\[
\begin{align*}
\text{ztree } z \text{ nil Empty } & \triangleq \mathbf{emp} \\
\text{ztree } z \ t \ (\text{Tip } \alpha) & \triangleq t \mapsto 0, \alpha \\
\text{ztree } z \ t \ (\text{Node } \lambda \ \rho) & \triangleq \exists l, r \cdot (t \mapsto 1, l, r \ast \text{ztree } z \ l \ \lambda \ast \text{ztree } z \ r \ \rho)
\end{align*}
\]

- ztree 0.5 \ t \ (\text{Node } (\text{Tip } 4) \ (\text{Tip } 4)) isn’t necessarily separated: it could be a DAG.
We do have some unsolved problems ...

\[ \text{ztree } z \text{ nil Empty } \overset{\cong}{=} \textbf{emp} \]
\[ \text{ztree } z \ t \ (\text{Tip } \alpha) \overset{\cong}{=} t \overset{z}{\mapsto} 0, \alpha \]
\[ \text{ztree } z \ t \ (\text{Node } \lambda \, \rho) \overset{\cong}{=} \exists \ l, \ r \cdot (t \overset{z}{\mapsto} 1, l, r \ast \text{ztree } z \ l \ \lambda \ast \text{ztree } z \ r \ \rho) \]

- \text{ztree } 0.5 \ t \ (\text{Node } (\text{Tip } 4) \ (\text{Tip } 4)) \text{ isn’t necessarily separated: it could be a DAG.} \]
- \text{We have two models – fractions and counting. Both seem to be necessary at present (maybe ...).}
Future work
Future work

- Variables as resources.
Future work

- Variables as resources.
- Existence permissions.
Future work

- Variables as resources.
- Existence permissions.
- Semaphores in the heap.
Future work

- Variables as resources.
- Existence permissions.
- Semaphores in the heap.
- Soundness (we just need a nod ...)
Accounting for variables
Accounting for variables

\[
\frac{\{Q\} C\{R\}}{\{P \star Q\} C\{P \star R\}} \quad (\text{modifies } C \cap \text{vars } P = \emptyset).
\]
Accounting for variables

\[
\int_{Q} \mathcal{C} \int_{R} \quad (\text{modifies } C \cap \text{vars } P = \emptyset).
\]

\[
\int_{P \star Q} \mathcal{C} \int_{P \star R}
\]

\[
\begin{align*}
\{ \text{emp} \} \\
\{ \text{own} (\text{count}) \land \ldots \} \\
\text{if } \text{count} = 0 \text{ then } \ldots \quad \text{// safe: I can read } \text{count} \\
\{ \text{own} (\text{count}) \land z \xrightarrow{\text{count}} N \} \\
\text{count} + := 1; \quad \text{// safe: I can read and write } \text{count} \\
\{ \text{own} (\text{count}) \land z \xrightarrow{\text{count}} N \star z \mapsto N \} \\
\text{V(} \text{read} \text{)} \\
\{ z \mapsto N \} \\
\quad \ldots \quad \text{// no longer safe to read or write } \text{count}
\end{align*}
\]
Accounting for variables

\[
\begin{align*}
\{Q\} C \{R\} \\
\{P \star Q\} C \{P \star R\}
\end{align*}
\]

(modifies \(C \cap \text{vars } P = \emptyset\)).

\[
\begin{align*}
\{\text{emp}\} \\
P(read); \\
\{\text{own}(count) \land \ldots\}
\end{align*}
\]

if \(count = 0\) then \ldots // safe: I can read \(\text{count}\)

\[
\begin{align*}
\{\text{own}(count) \land z \overset{\text{count}}{\rightarrow} N\}
\end{align*}
\]

\(count + := 1\); // safe: I can read and write \(\text{count}\)

\[
\begin{align*}
\{\text{own}(count) \land z \overset{\text{count}}{\rightarrow} N \star z \hookrightarrow N\}
\end{align*}
\]

\(V(\text{read})\)

\[
\begin{align*}
\{z \hookrightarrow N\}
\end{align*}
\]

... // no longer safe to read or write \(\text{count}\)

Imagine: no non-interference side-conditions; no anti-aliasing side-condition on variable assignment; elegant proofs of Dijkstra’s semaphore programs – now that’s hubris!
Is anybody there?
Is anybody there?

- A semaphore has permission to read and write its variable; a user has permission to $P$ and $V$ it.
A semaphore has permission to read and write its variable; a user has permission to P and V it.

So long as any user has that permission, the semaphore can’t dispose itself.
A semaphore has permission to read and write its variable; a user has permission to P and V it.

So long as any user has that permission, the semaphore can’t dispose itself.

Existence permissions give no access to resource contents.
A semaphore has permission to read and write its variable; a user has permission to P and V it.

So long as any user has that permission, the semaphore can’t dispose itself.

Existence permissions give **no** access to resource contents.

Proof theory looks easy, but no agreed model yet.
Last one to leave, turn off the light!
Pipe-line processing fits permission counting: e.g. multicasting in a network processor.
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Each packet-buffer has a semaphore, in neighbouring heap location, which counts permissions.
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‘start’ gives you a buffer and a 1 semaphore.
Last one to leave, turn off the light!

- Pipeline processing fits permission counting: e.g. multicasting in a network processor.
- Each packet-buffer has a semaphore, in neighbouring heap location, which counts permissions.
- ‘start’ gives you a buffer and a 1 semaphore.
- ‘split’ V s the semaphore, and releases an extra read permission.
Pipeline processing fits permission counting: e.g. multicasting in a network processor.

Each packet-buffer has a semaphore, in neighbouring heap location, which counts permissions.

‘start’ gives you a buffer and a 1 semaphore.

‘split’ Vs the semaphore, and releases an extra read permission.

‘finish’ at the end of a pipeline Ps the semaphore – and if now 0, disposes semaphore and buffer together.
Pipeline processing fits permission counting: e.g. multicasting in a network processor.

Each packet-buffer has a semaphore, in neighbouring heap location, which counts permissions.

‘s’ start’ gives you a buffer and a 1 semaphore.

‘split’ Vs the semaphore, and releases an extra read permission.

‘finish’ at the end of a pipeline Ps the semaphore – and if now 0, disposes semaphore and buffer together.

The semaphore has to be hidden; it might need a CCR; it might need the hypothetical frame rule; it’s a semaphore in the heap!...
Soundness

We think we only need a nod.
Thank you!

John Tang Boyland, for pointing out that $\frac{1}{2} + \frac{1}{2} = 1$.

The East London Massive for listening to countless versions, and picking holes in them.

Steve Brookes, for taking us seriously.

Josh Berdine, John Reynolds and Hongseok Yang, for ruthless and relentless criticism.