# Comparison of Validity Indexes for Fuzzy Clusters of fMRI data

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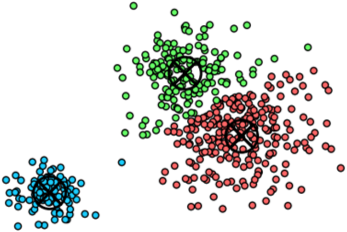
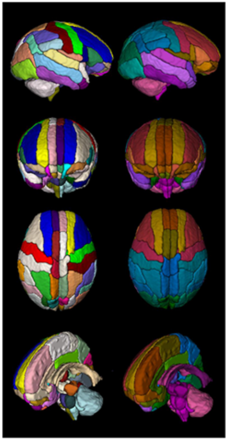
# Contents

# *functional* MRI
# Image Visualization and Processing





# *functional* MRI
## Resting-state and data-driven methods

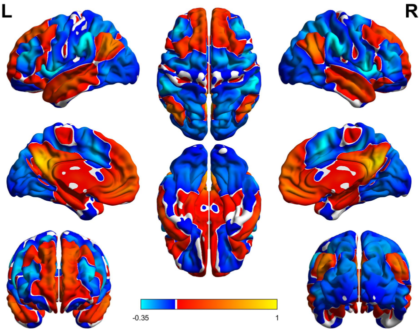



Figure: Example. This image displays **resting-state functional connectivity** as linear correlation for the seed region (Precuenus) in a sample of 1,000 subjects

# Clustering Approaches

### Clustering

Let $X = \{x_1, x_2, \ldots, x_n\}$ a given **dataset** (with $n$ elements), and let $C = \{c_1, c_2, ..., c_K\}$ the set of **cluster**, where $K$ is the desired number of clusters.

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Regardless of the criterion chosen for the partition, the purpose of clustering is to develop a **partition matrix** of size $K \times n$ denoted as $U = [\mu_{ij}]$, with $i = 1, 2, \ldots, K$ and $j = 1, 2, \ldots, n$, where $\mu_{ij}$ is the grade of membership of point $x_j$ to cluster $c_i$.

# Clustering Approaches

### Crisp case (classic logic, $\mathbb{Z}$)

In crisp clustering, each point in the specified dataset belongs to a **single cluster class**. Then $\mu_{ij} = 1$ if $x_j \in c_i$, otherwise $\mu_{ij} = 0$.

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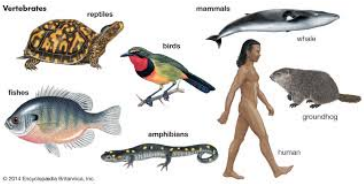
### Fuzzy case (many-valued logic, $\mathbb{R}$)

In fuzzy clustering, a point can be associated with **multiple clusters** with a certain **degree of membership**, and the partition matrix in this case is represented as $U = [\mu_{ij}]$, where $\mu_{ij} \in [0, 1]$ indicates the degree of membership of the $j$-th element to the $i$-th cluster.

# Example of Crisp Partition Matrix
# Which category is an animal?



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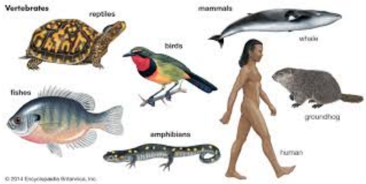


| Animals | Mammals | Amphibians | Birds |
|---|---|---|---|
| Dog | 1 | 0 | 0 |
| Frog | 0 | 1 | 0 |
| Coyote | 1 | 0 | 0 |
| Opossum | 1 | 0 | 0 |
| Eagle | 0 | 0 | 1 |

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# Is a bunch of granes an heap?

| granes | heap | no heap |
|---|---|---|
| 1 | 0 | 1 |
| 100 | 0.3 | 0.7 |
| 1000 | 0.5 | 0.5 |
| 10000 | 0.75 | 0.25 |
| 100000 | 0.9 | 0.1 |

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**Given a predefined number of classes**, the purpose of the FCM algorithm is to create vectors called **centroids** that minimize the value of the function $J_m$ that is given by the sum of the intra-cluster quadratic error. $J_m$ it is defined as:

$$J_m = \sum_{j=1}^{n} \sum_{i=1}^{K} \mu_{ij}^{m} ||x_j - z_i||^2 \tag{1}$$

$\mathbf{m} > 1$ is the **exponent** to adjust the degree of fuzzy overlap.

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$\longrightarrow$ how to compare several indexes?

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Given a **type** of clustering algorithm, CVIs compute a **relation** between Compactness C and Separation S, and use a **criterion** to determine the optimum.

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- type: crisp or fuzzy
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1. DBI: C $\backslash$ S | *min* | crisp
2. SDBI: C $\backslash$ S | *min* | fuzzy
3. XBI: C $\backslash$ S | *min* | fuzzy
4. PBMI: C $\times$ S | *max* | crisp
5. FPBMI: C $\times$ S | *max* | fuzzy
6. RLRI: C $+$ S | *min* | fuzzy
7. WSJI: C $+$ S | *min* | fuzzy
8. FSI: C $-$ S | *min* | fuzzy

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### Configurations

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$\longrightarrow$ How much should we **rely** on the optimality measure of an index?
$\longrightarrow$ Are there **limits** on the optimality criterion?

# Computational Experiments

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$$E = |ni - nr| \tag{2}$$

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- **2 fMRI classes**:
  - yes/no of ROIs about the Default Mode Network (**DMN**)
- **4 fMRI classes** associated to
  - Default Mode Network (**DMN**)
  - Visual Network (**VN**)
  - Sensory/Motor Network **(SMN)**
  - Other Resting Networks (**ORN**)

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*Rescale* and *costrain* the direction: the lower, the better

$$Z(* \longrightarrow min) = +\frac{x - \mu}{\sigma} \quad \text{and} \quad Z(* \longrightarrow max) = -\frac{x - \mu}{\sigma} \tag{3}$$

# Comparison Results

Table: Error of the **8 indexes** evaluating clustering with 2 and 4 references

| Index | Experiment 1 | | Index | Experiment2 | |
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| SDBI (/) | 1.54 | 1.50 | SDBI (/) | 1.55 | 0.49 |
| DBI (/) | 4.76 | 5.02 | DBI (/) | 3.31 | 2.35 |
| XBI (/) | 5.50 | 0.40 | XBI (/) | 3.69 | 0.39 |
| PBMI (×) | 6.22 | 0.42 | PBMI (×) | 4.22 | 0.42 |
| FPBMI (×) | 6.36 | 0.25 | FPBMI (×) | 4.36 | 0.25 |

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- Add **other measures** in addition to $E = |ni - nr|$
- Use a **fuzzy inference system** to merge the measures

# Cheers

Thank you for the attention!

## Reference for other details

Martinelli et al 2019
Comparison of Validity Indexes for Fuzzy Clusters of fMRI Data
*In: Tavares J., Natal Jorge R. (eds) VipIMAGE 2019. VipIMAGE 2019. Lecture Notes in Computational Vision and Biomechanics, vol 34. Springer, Cham*